Appendix: Proofs

Proof of Lemma 1

Proof. The function $g(r)$ is continuous and strictly increasing in the interval $[0, \infty)$, its derivative satisfies $g'(r) = \frac{t\theta}{N(r+\theta)^2} > 0$ for all $r \geq 0$. In addition, it is easy to show that $g(0) = c$ and $\lim g(r) = c + \frac{t}{N}$ as $r \to \infty$. Then, there must exist a unique r^* such that $g(r^*) = r^*$ that satisfies $c < r^* < c + \frac{t}{N}$. This completes the proof.

Proof of Proposition 1

Proof. For a symmetric equilibrium of the model, there are three relevant cases for equilibrium prices for any given reference price r, i.e., $p^* > r$, $p^* < r$, and $p^* = r$. The FOC of this maximization problem satisfies the following expression:

$$(p_i - c)\frac{\partial d_i}{\partial p_i} + \left(\frac{1}{N} + \frac{p - p_i}{t} + \frac{\theta}{tr}\left[max\{p - r, 0\} - max\{p_i - r, 0\}\right]\right) = 0 \tag{15}$$

where $\frac{\partial d_i}{\partial p_i} = -\left[\frac{1}{t} + \frac{\theta}{tr}\left(\frac{1+sign(p_i-r)}{2}\right)\right]$ and:

$$sign\,(x) = \begin{cases} 1 \quad if \quad x > 0 \\ -1 \quad if \quad x < 1 \end{cases}$$

Then the best response function of firm i satisfies the following equation:

$$BR_i\,(p) = \begin{cases} \frac{c}{2} + \frac{tr}{2N(r+\theta)} + \frac{pr}{2(r+\theta)} + \frac{\theta}{2(r+\theta)}\left(max\left\{p-r,0\right\}+r\right) \\ if \quad p_i > r \\ \frac{c}{2} + \frac{t}{2N} + \frac{p}{2} + \frac{\theta}{2r} max\left\{p-r,0\right\} if \quad p_i < r \end{cases} \tag{16}$$

Hence, the FOC is useful to characterize the first two relevant cases for the equilibrium price. We know that, in a symmetric equilibrium, $p_i = p = p^*$. Based on the best response function of firms, it is easy to show that in the first relevant case of the proof, the equilibrium price must satisfy $p^* = c + \frac{tr}{N(r+\theta)}$ whenever $p^* > r$. According to Lemma 1, this condition holds whenever $r^* > r$, where r^* is the unique reference price that satisfies $r^* = c + \frac{tr^*}{N(r^*+\theta)}$. For the second case, the equilibrium price satisfies $p^* = c + \frac{t}{N}$ whenever $c + \frac{t}{N} > r$.

For the third relevant case, i.e., $p^* = r$, we must implement a direct proof since the best response function of firm i is not well-defined at this point. In general, we have to prove that playing r is the best response of the firm i whenever all other firms play symmetrically $p = r$. Suppose that all firms, except for the firm i, play a price $p = r$ that satisfies $r^* \leq r \leq c + \frac{t}{N}$. There are two cases to be analyzed: $p_i > r$ and $p_i < r$, for a proper $\varepsilon > 0$, respectively. Consider the first case, it is clear that profits of firm i can be written in the following way:

$$(r+\varepsilon-c)\left(\frac{1}{N} - \frac{\varepsilon}{t} - \frac{\theta\varepsilon}{tr}\right) = (r-c)\,\frac{1}{N}$$

$$+\left[1-(r-c)\left(\frac{(r+\theta)\,N}{tr}\right)\right]\frac{\varepsilon}{N} - \left(\frac{r+\theta}{tr}\right)\varepsilon^2 \tag{17}$$

where $(r-c)\frac{1}{N}$ is the profit that is obtained by following the strategy $p_i = r$. Since $r^* \leq r \leq c + \frac{t}{N}$, there must exist $\bar{r} \geq r$ such that $r = c + \frac{t\bar{r}}{N(\bar{r}+\theta)}$. By substituting in equation (17) and by noting that $\frac{tr}{N(r+\theta)}$ is a continuous and strictly increasing function in r, we find that:

$$(r-c)\frac{1}{N} + \left[1 - \frac{\frac{t\bar{r}}{N(\bar{r}+\theta)}}{\frac{tr}{N(r+\theta)}}\right]\frac{\varepsilon}{N} - \left(\frac{r+\theta}{tr}\right)\varepsilon^2 < (r-c)\frac{1}{N} \qquad (18)$$

Following a similar argument, in the case where $p_i < r$ profits of firm i can be written as:

$$(r-\varepsilon-c)\left(\frac{1}{N}+\frac{\varepsilon}{t}\right) = (r-c)\frac{1}{N} + \left[(r-c)\frac{N}{t} - 1\right]\frac{\varepsilon}{N} - \frac{\varepsilon^2}{t} \quad (19)$$

As before, by substituting $\bar{r} \geq r$ such that $r = c + \frac{t\bar{r}}{N(\bar{r}+\theta)}$ in equation (19) we find that:

$$(r-c)\frac{1}{N} + \left[\frac{\bar{r}}{\bar{r}+\theta} - 1\right]\frac{\varepsilon}{N} - \frac{\varepsilon^2}{t} < (r-c)\frac{1}{N} \qquad (20)$$

Hence, playing $p_i = r$ is a best response for firm i whenever all other firms are playing $p = r$. Then, the equilibrium market price is given by the function:

$$p^*(r) = \begin{cases} c + \frac{tr}{N(r+\theta)} & if \quad 0 \leq r < r^* \\ r & if \quad r^* \leq r \leq c + \frac{t}{N} \\ c + \frac{t}{N} & if \quad c + \frac{t}{N} < r \end{cases} \qquad (21)$$

This completes the proof.

Proof of Proposition 2

Proof. Assume that $0 < r < r^{**}$, then the derivative of $CS(r)$ satisfies:

$$\frac{\partial CS\left(r\right)}{\partial r}=-\frac{\theta}{r^{2}}\left[\frac{1}{2}\sqrt{\frac{r+\theta}{r}}\left(\frac{\theta r}{\left(r+\theta\right)^{2}}\right)\sqrt{tF}-c-\sqrt{\frac{r}{r+\theta}}\sqrt{tF}\right]$$

$$-\frac{1}{2}\sqrt{\frac{r+\theta}{r}}\left(\frac{\theta}{\left(r+\theta\right)^{2}}\right)\sqrt{tF}+\frac{1}{8}\sqrt{\frac{r}{r+\theta}}\left(\frac{\theta}{r^{2}}\right)\sqrt{tF} \quad (22)$$

By rearranging the previous expression, it is possible to show that equation (22) satisfies:

$$\frac{\partial CS\left(r\right)}{\partial r}=\frac{c\theta}{r^{2}}-\frac{c\theta}{2r^{2}}\sqrt{\frac{r}{r+\theta}}\left(\frac{\theta}{r+\theta}-2\right)\sqrt{tF}$$

$$-\frac{c\theta}{2r^{2}}\sqrt{\frac{r}{r+\theta}}\left(\frac{r}{r+\theta}-\frac{1}{4}\right)\sqrt{tF} \quad (23)$$

which reduces to the following expression:

$$\frac{\partial CS\left(r\right)}{\partial r}=\frac{c\theta}{r^{2}}\left(1+\frac{5}{8}\sqrt{\frac{r}{r+\theta}}\sqrt{tF}\right)>0 \quad (24)$$

Then, the consumer surplus is always strictly increasing for $0<r<r^{**}$. Now consider the case where $r^{**}\leq r\leq c+\sqrt{tF}$, in this case the derivative of consumer surplus satisfies:

$$\frac{\partial CS\left(r\right)}{\partial r}=-1+\frac{tF}{4\left(r-c\right)^{2}} \quad (25)$$

Since $r^{**}>c$, this derivative is well defined and the $CS(r)$ attains a maximum at the reference price $\hat{r}=c+\frac{1}{2}\sqrt{tF}$ and $CS\left(\hat{r}\right)=U-c-\sqrt{tF}>U-c-\frac{5}{4}\sqrt{tF}$. In addition, note that there are two real roots $r1>0$ and $r2>0$ that satisfy the condition $U-r-\frac{1}{4}\left(\frac{tF}{r-c}\right)=U-c-\frac{5}{4}\sqrt{tF}$. Given that, it is possible to show that $r1=c+\frac{1}{4}\sqrt{tF}$ and $r2=c+\sqrt{tF}$. Since $CS(r)$ is a continuous function at the point r^{**}, the following condition is satisfied:

$$U - \theta max \left\{ \frac{c + \sqrt{\frac{r^{**}}{r^{**}+\theta}}\sqrt{tF} - r^{**}}{r^{**}}, 0 \right\} - c - \sqrt{\frac{r^{**}}{r^{**}+\theta}}\sqrt{tF}$$

$$-\frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{tF} = U - r^{**} - \frac{1}{4}\left(\frac{tF}{r^{**}-c}\right) \tag{26}$$

This implies that $CS\left(r^{**}\right) > U - c - \frac{5}{4}\sqrt{tF}$ for all all $r1 < r^{**} < r2$. Hence, there are two candidates for being the global argmax of the function $CS(r)$, depending on the values of parameters. Accordingly, $\hat{r}$ would be the argmax of $CS(r)$ only if it is on the right side of r^{**}, otherwise the global argmax will be r^{**}. Hence, $\hat{r} = c + \frac{1}{2}\sqrt{tF} \geq c + \sqrt{\frac{r^{**}}{r^{**}+\theta}}\sqrt{tF} = r^{**}$ whenever $\frac{1}{4} \geq \frac{r^{**}}{r^{**}+\theta}$, which is equivalent to $r^{**} \leq \frac{\theta}{3}$. This completes the proof.

Proof of Proposition 3

Proof. For the case of $r^{**} \leq \frac{\theta}{3}$, the previously mentioned properties are trivially satisfied, given that r^{***} takes the closed form solution of $c + \frac{1}{2}\sqrt{tF}$. Since the optimal reference price has no closed form solution when $r^{**} > \frac{\theta}{3}$, we can calculate the implicit derivatives from the function $r = c + \sqrt{\frac{r}{r+\theta}}\sqrt{tF}$ concerning each parameter of interest. In order to simplify, with some abuse of notation, we use r instead of r^{**} to indicate the optimal reference price in this case.

Case 1: By implicitly differentiating $r = c + \sqrt{\frac{r}{r+\theta}}\sqrt{tF}$ with respect to c, we attain the following expression:

$$\frac{\partial r}{\partial c} = 1 + \frac{1}{2}\sqrt{\frac{r+\theta}{r}}\sqrt{tF}\left(\frac{(r+\theta)\frac{\partial r}{\partial c} - r\frac{\partial r}{\partial c}}{(r+\theta)^2}\right) \tag{27}$$

By rearranging equation (27), we obtain:

$$\frac{\partial r}{\partial c} = 1 + \frac{1}{2}\sqrt{\frac{r}{r+\theta}}\sqrt{tF}\left(\frac{1}{r+\theta}\right)\left(\frac{\theta}{r}\right)\frac{\partial r}{\partial c} \tag{28}$$

Since, by definition, $r - c = \sqrt{\frac{r}{r+\theta}}\sqrt{tF}$, equation (28) can be reduced to:

$$\frac{\partial r}{\partial c} = \frac{1}{1 - \frac{1}{2}\left(\frac{r-c}{r}\right)\left(\frac{\theta}{r+\theta}\right)} > 0 \tag{29}$$

Case 2: Similarly, by implicitly differentiating the reference price function with respect to t, we obtain:

$$\frac{\partial r}{\partial t} = \frac{1}{2}\sqrt{\frac{r+\theta}{r}}\sqrt{tF}\left(\frac{(r+\theta)\frac{\partial r}{\partial t} - r\frac{\partial r}{\partial t}}{(r+\theta)^2}\right) + \frac{1}{2}\sqrt{\frac{r}{r+\theta}}\sqrt{\frac{F}{t}} \tag{30}$$

By rearranging this equation and substituting $r - c = \sqrt{\frac{r}{r+\theta}}\sqrt{tF}$ in equation (30), we have that:

$$\frac{\partial r}{\partial t} = \frac{\frac{1}{2}\left(\frac{r-c}{t}\right)}{1 - \frac{1}{2}\left(\frac{r-c}{r}\right)\left(\frac{\theta}{r+\theta}\right)} > 0 \tag{31}$$

Case 3: Basically, the same procedure as for t. Simply exchange t by F in the previous implicit derivative.

Case 4: By implicitly differentiating the reference price function with respect to θ, we obtain:

$$\frac{\partial r}{\partial \theta} = \frac{1}{2}\sqrt{\frac{r+\theta}{r}}\sqrt{tF}\left(\frac{(r+\theta)\frac{\partial r}{\partial \theta} - r\left(\frac{\partial r}{\partial \theta} + 1\right)}{(r+\theta)^2}\right) \tag{32}$$

By rearranging equation (31), we can express equation (32) as follows:

$$\frac{\partial r}{\partial \theta} = -\frac{\frac{1}{2}\left(\frac{r-c}{r+\theta}\right)}{1 - \frac{1}{2}\left(\frac{r-c}{r}\right)\left(\frac{\theta}{r+\theta}\right)} < 0 \tag{33}$$

This completes the proof.

Proof of Proposition 4

Proof. Let us consider the function that characterizes the number of firms given by the expression $N(r) = \frac{r-c}{F}$ for $r^{**} \leq r < c + \sqrt{tF}$.

Case 1: By directly differentiating $N(r)$ with respect to c, we obtain:

$$\frac{\partial N(r)}{\partial c} = \frac{1}{F}\left(\frac{\partial r^{***}}{\partial c} - 1\right) \tag{34}$$

For the case of $r^{**} > \frac{\theta}{3}$, equation (34) is equivalent to the following:

$$\frac{\partial N(r^{***})}{\partial c} = \frac{1}{F}\left(\frac{\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}{1 - \frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}\right) \tag{35}$$

This equation is positive, since $1 - \frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right) > 0$. For the case of $r^{**} \leq \frac{\theta}{3}$, we know that $\frac{\partial r^{***}}{\partial c} = 1$.

Case 2: In a similar way, by differentiating $N(r)$ with respect to t, we have that:

$$\frac{\partial N(r^{***})}{\partial t} = \frac{1}{F}\frac{\partial r^{***}}{\partial t} \tag{36}$$

Hence, $\frac{\partial N(r^{***})}{\partial t} > 0$, since $\frac{\partial r^{***}}{\partial t} > 0$.

Case 3: By directly differentiating $N(r)$ with respect to F, we obtain:

$$\frac{\partial N(r)}{\partial F} = \frac{1}{F}\left(\frac{\partial r^{***}}{\partial F} - \frac{r^{***}-c}{F}\right) \tag{37}$$

We know that $\frac{\partial r^{***}}{\partial F} = \frac{\frac{1}{2}\left(\frac{r^{***}-c}{F}\right)}{1-\frac{1}{2}\left(\frac{r^{***}-c}{F}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}$ for the case of $r^{**} > \frac{\theta}{3}$. Hence, equation (37) is equivalent to the following:

$$\frac{\partial N(r^{***})}{\partial F} = -\frac{1}{F}\left(\frac{\frac{1}{2} - \frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}{1 - \frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}\right) \tag{38}$$

It is clear that $1-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)>\frac{1}{2}$, then $\frac{1}{2}-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)>0$, which implies that $\frac{\partial N(r^{***})}{\partial F}<0$. When $r^{**}\leq\frac{\theta}{3}$, the optimal reference price satisfies $r^{***}=c+\frac{1}{2}\sqrt{tF}$, which implies that $\frac{\partial r^{***}}{\partial F}=\frac{1}{4}\sqrt{\frac{t}{F}}$ and $\frac{r^{***}-c}{F}=\frac{1}{2}\sqrt{\frac{t}{F}}$, hence $\frac{\partial N(r^{***})}{\partial F}<0$.

Case 4: For the last case, by differentiating $N(r)$ with respect to θ, we have the expression:

$$\frac{\partial N\left(r^{***}\right)}{\partial\theta}=\frac{1}{F}\frac{\partial r^{***}}{\partial\theta} \tag{39}$$

which directly implies the result, since we know that $\frac{\partial(r^{***})}{\partial\theta}<0$ whenever $r^{**}>\frac{\theta}{3}$ and $\frac{\partial r^{***}}{\partial\theta}=0$ whenever $r^{**}\leq\frac{\theta}{3}$. This completes the proof.

Proof of Proposition 5

Proof. Let us consider the transportation cost function that is relevant for the analysis given by $TC\left(r\right)=\frac{1}{4}\left(\frac{tF}{r-c}\right)$ for $r^{**}\leq r<c+\sqrt{tF}$.

Case 1: By directly differentiating $TC(r)$ with respect to c, we obtain:

$$\frac{\partial TC\left(r^{***}\right)}{\partial c}=-\frac{tF}{4{(r^{***}-c)}^{2}}\left(\frac{\partial r^{***}}{\partial c}-1\right) \tag{40}$$

Since for the case of $r^{**}>\frac{\theta}{3}$ we know that $\frac{\partial r^{***}}{\partial c}>1$, this implies that $\frac{\partial TC(r^{***})}{\partial c}<1$. For the case of $r^{**}\leq\frac{\theta}{3}$, we know that $\frac{\partial r^{***}}{\partial c}=1$, hence $\frac{\partial TC(r^{***})}{\partial c}=0$.

Case 2: In a similar way, by differentiating $TC(r)$ with respect to t, we have the following:

$$\frac{\partial TC\left(r^{***}\right)}{\partial t}=\frac{\left(r^{***}-c\right)F-tF\frac{\partial r^{***}}{\partial t}}{4{(r^{***}-c)}^{2}} \tag{41}$$

After some manipulation, equation (41) reduces to the following:

$$\frac{\partial TC\left(r^{***}\right)}{\partial t}=\frac{tF}{4(r^{***}-c)^2}\left(\frac{r^{***}-c}{t}-\frac{\partial r^{***}}{\partial t}\right) \tag{42}$$

Given that $\frac{\partial r^{***}}{\partial t}=\frac{\frac{1}{2}\left(\frac{r^{***}-c}{t}\right)}{1-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}$ for $r^{**}>\frac{\theta}{3}$, , we know that equation (42) reduces to the following:

$$\frac{\partial TC\left(r^{***}\right)}{\partial t}=\frac{F}{4\left(r^{***}-c\right)}\left(\frac{\frac{1}{2}-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}{1-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}\right)>0 \tag{43}$$

This equation is positive, since $\frac{1}{2}-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)>0$. When $r^{**}\leq\frac{\theta}{3}$, the optimal reference price satisfies $r^{***}=c+\frac{1}{2}\sqrt{tF}$, which implies that $\frac{\partial r^{***}}{\partial t}=\frac{1}{4}\sqrt{\frac{F}{t}}$ and $\frac{r^{***}-c}{t}=\frac{1}{2}\sqrt{\frac{F}{t}}$, hence $\frac{\partial N(r^{***})}{\partial F}>0$.

Case 3: Basically, the same case as for t, by substituting t with F and vice versa. Hence, when $r^{**}>\frac{\theta}{3}$ the following condition is satisfied:

$$\frac{\partial TC\left(r^{***}\right)}{\partial F}=\frac{t}{4\left(r^{***}-c\right)}\left(\frac{\frac{1}{2}-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}{1-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}\right)>0 \tag{44}$$

and whenever $r^{**}\leq\frac{\theta}{3}$ we have:

$$\frac{\partial N\left(r^{***}\right)}{\partial F}=\frac{tF}{4(r^{***}-c)^2}\left(\frac{r^{***}-c}{F}-\frac{\partial r^{***}}{\partial F}\right)>0$$

Case 4: For the last case, by differentiating $TC(r)$ with respect to θ, we have the following expression:

$$\frac{\partial TC\left(r^{***}\right)}{\partial\theta}=-\frac{tF}{4(r^{***}-c)^2}\frac{\partial r^{***}}{\partial\theta} \tag{45}$$

which directly implies the result, since we know that $\frac{\partial r^{***}}{\partial \theta} < 0$ whenever $r^{**} > \frac{\theta}{3}$, and $\frac{\partial r^{***}}{\partial \theta} = 0$ whenever $r^{**} \leq \frac{\theta}{3}$. This completes the proof.

Proof of Proposition 6

Proof. For the case of $r^{**} > \frac{\theta}{3}$, the social welfare function is not differentiable at r^{**}, which is the argmax of the policymaker problem. In this case, it is only possible to determine the lower and upper bounds of the variation of the value function through the right-hand and the left-hand side partial derivatives of the objective function evaluated at the optimal reference price. Otherwise, whenever $r^{**} \leq \frac{\theta}{3}$, the objective function is differentiable at the optimal reference price r^{**}, and a regular envelope theorem can be applied.

Case 1: By differentiating the right-hand side of the social welfare function $CS(r, c, t, F, \theta)$ with respect to c, we obtain:

$$\frac{\partial CS\left(\cdot\right)}{\partial c} = -\frac{tF}{4(r-c)^2} \tag{46}$$

Since $r^{**} = c + \sqrt{\frac{tFr^{**}}{r^{**}+\theta}}$ for $r^{**} > \frac{\theta}{3}$, after evaluating $\frac{\partial CS(\cdot)}{\partial c}$, we have that:

$$\frac{\partial CS\left(\cdot\right)}{\partial c} = -\frac{r^{**}+\theta}{4r^{**}} \tag{47}$$

Similarly, by differentiating the left-hand side of the social welfare function and evaluating at $r^{**} = c+\sqrt{\frac{tFr^{**}}{r^{**}+\theta}}$, we obtain the following:

$$\frac{\partial CS\left(\cdot\right)}{\partial c} = -\frac{r^{**}+\theta}{r^{**}} \tag{48}$$

Hence, the variation of the social welfare function in the face of an increase in the marginal cost $\frac{\partial CS(r^{***},c,t,F,\theta)}{\partial c}$ must be in the interval $\left[-\frac{r^{**}+\theta}{r^{**}}, -\frac{r^{**}+\theta}{4r^{**}}\right]$ whenever $r^{**} > \frac{\theta}{3}$. For the case in which $r^{**} \leq \frac{\theta}{3}$, the partial derivative of the value function satisfies the equation (34). Evaluating this expression at the optimal reference price $r^{***} = c + \frac{1}{2}\sqrt{tF}$ implies that:

$$\frac{\partial CS\left(r^{***}, c, t, F, \theta\right)}{\partial c} = -1 \tag{49}$$

Case 2: By differentiating the right-hand side of the social welfare function $CS(r,c,t,F,\theta)$ with respect to t, we obtain

$$\frac{\partial CS\left(\cdot\right)}{\partial t}=-\frac{F}{4(r-c)} \tag{50}$$

Since $r^{**}=c+\sqrt{\frac{tFr^{**}}{r^{**}+\theta}}$ for $r^{**}>\frac{\theta}{3}$, after evaluating $\frac{\partial CS(\backslash cdotp)}{\partial t}$ atr^{**}, we have that:

$$\frac{\partial CS\left(\cdot\right)}{\partial t}=-\frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}} \tag{51}$$

Similarly, by differentiating the left-hand side of the social welfare function, we obtain:

$$\frac{\partial CS\left(\cdot\right)}{\partial t}=-\frac{\theta}{2r}\sqrt{\frac{r}{r+\theta}}\sqrt{\frac{F}{t}}-\frac{1}{2}\sqrt{\frac{r}{r+\theta}}\sqrt{\frac{F}{t}}-\frac{1}{8}\sqrt{\frac{r+\theta}{r}}\sqrt{\frac{F}{t}} \tag{52}$$

After simplifying and evaluating at r^{**}, equation (52) reduces to the following:

$$\frac{\partial CS\left(\cdot\right)}{\partial t}=-\frac{5}{8}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}} \tag{53}$$

Hence, the variation of the social welfare function in the face of an increase in the transportation cost $\frac{\partial CS(r^{***},c,t,F,\theta)}{\partial t}$ must be in the interval $\left[-\frac{5}{8}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}},-\frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}}\right]$ whenever $r^{**}>\frac{\theta}{3}$. For the case in which $r^{**}\leq\frac{\theta}{3}$, the partial derivative of the value function satisfies equation (50). Evaluating this expression at the optimal reference price $r^{***}=c+\frac{1}{2}\sqrt{tF}$ implies that:

$$\frac{\partial CS\left(r^{***},c,t,F,\theta\right)}{\partial t}=-\frac{1}{2}\sqrt{\frac{F}{t}} \tag{54}$$

Case 3: The case of F is identical to the one of t. It suffices to exchange t with F and vice versa. Hence, $\frac{\partial CS(r^{***},c,t,F,\theta)}{\partial F}$ must be in the interval $\left[-\frac{5}{8}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{t}{F}},-\frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{t}{F}}\right]$ whenever $r^{**}>\frac{\theta}{3}$, and $\frac{\partial CS(r^{***},c,t,F,\theta)}{\partial F}=-\frac{1}{2}\sqrt{\frac{t}{F}}$ whenever $r^{**}\leq\frac{\theta}{3}$.

Case 4: By right-hand side differentiating the social welfare function $CS(r, c, t, F, \theta)$ with respect to θ, we obtain:

$$\frac{\partial CS(\cdot)}{\partial \theta} = 0 \tag{55}$$

Similarly, by differentiating the left-hand side of the social welfare function with respect to θ, we obtain the following:

$$\frac{\partial CS(\cdot)}{\partial \theta} = \frac{\theta}{2r}\sqrt{\frac{r}{r+\theta}}\left(\frac{\sqrt{tF}}{r+\theta}\right) - \left(\frac{c + \sqrt{\frac{tFr}{r+\theta}} - r}{r}\right)$$

$$+\frac{1}{2}\sqrt{\frac{r}{r+\theta}}\left(\frac{\sqrt{tF}}{r+\theta}\right) - \frac{1}{8}\sqrt{\frac{r}{r+\theta}}\left(\frac{\sqrt{tF}}{r}\right) \tag{56}$$

After simplifying and evaluating at r^{**}, equation (56) reduces to the following:

$$\frac{\partial CS(\cdot)}{\partial \theta} = \frac{1}{2}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{tF}\left(\frac{1}{r^{**}+\theta} - \frac{1}{4r^{**}}\right) > 0 \tag{57}$$

Hence, the variation of the social welfare function in the face of an increase in the parameter θ, $\frac{\partial CS(r^{***},c,t,F,\theta)}{\partial \theta}$ must be in the interval $\left[0, \frac{1}{2}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{tF}\left(\frac{1}{r^{**}+\theta} - \frac{1}{4r^{**}}\right)\right]$ whenever $r^{**} > \frac{\theta}{3}$. For the case in which $r^{**} \leq \frac{\theta}{3}$, the partial derivative of the value function satisfies equation (55). Evaluating this expression at the optimal reference price $r^{***} = c + \frac{1}{2}\sqrt{tF}$ implies that:

$$\frac{\partial CS(r^{***}, c, t, F, \theta)}{\partial \theta} = 0 \tag{58}$$

This completes the proof.

Proof of Corollary 2

Proof. The proof of this Corollary is essentially based on the proof of Proposition 6. Note that the difference in consumer welfare is defined as $\Delta CS(r,c,t,F,\theta) = CS(r,c,t,F,\theta) - \left(U - c - \frac{5}{4}\sqrt{tF}\right)$. Hence, the second part of this expression is simply the consumer welfare of a model with no reference prices, so that the function $\widehat{CS} = U - c - \frac{5}{4}\sqrt{tF}$ is independent of reference prices and their right-hand and the left-hand side partial derivatives coincide and are equal to their corresponding partial derivatives. Following the previous argument, it is easy to show that partial derivatives of the function $\widehat{CS}$ are equal to $\frac{\partial \widehat{CS}(\cdot)}{\partial c} = -1$, $\frac{\partial \widehat{CS}(\cdot)}{\partial t} = -\frac{5}{8}\sqrt{\frac{F}{t}}$, $\frac{\partial \widehat{CS}(\cdot)}{\partial F} = -\frac{5}{8}\sqrt{\frac{t}{F}}$ and $\frac{\partial \widehat{CS}(\cdot)}{\partial \theta} = 0$. Following the proof of Proposition 6 and the previous observation about partial derivatives of $\widehat{CS}$, we can establish the following cases.

Case 1: Assume that $r^{**} > \frac{\theta}{3}$, by differentiating the right-hand side of the difference in social welfare function $\Delta CS(r,c,t,F,\theta)$ with respect to c and evaluating it at r^{**}, we obtain:

$$\frac{\partial \Delta CS(\cdot)}{\partial c} = 1 - \frac{r^{**}+\theta}{4r^{**}} \tag{59}$$

Similarly, by the left-hand side differentiating the difference in social welfare function and evaluating it at r^{**}, we obtain the following:

$$\frac{\partial \Delta CS(\cdot)}{\partial c} = 1 - \frac{r^{**}+\theta}{r^{**}} \tag{60}$$

Hence, the variation of the difference in social welfare function in the face of an increase in the marginal cost must be in the interval $\left[1 - \frac{r^{**}+\theta}{r^{**}}, 1 - \frac{r^{**}+\theta}{4r^{**}}\right]$ whenever $r^{**} > \frac{\theta}{3}$. For the case in which $r^{**} \leq \frac{\theta}{3}$, the variation of $\Delta CS(r,c,t,F,\theta)$ is equal to:

$$\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial c} = 0 \tag{61}$$

Case 2: By differentiating the right-hand side of the function ΔCS (r,c,t,F,θ) with respect to t and evaluating it at r^{**}, we obtain the following:

$$\frac{\partial \Delta CS(\cdot)}{\partial t} = \frac{5}{8}\sqrt{\frac{F}{t}} - \frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}} \tag{62}$$

Similarly, by differentiating the left-hand side of the social welfare function, we obtain:

$$\frac{\partial \Delta CS\left(\cdot\right)}{\partial t}=\frac{5}{8}\sqrt{\frac{F}{t}}-\frac{5}{8}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}} \tag{63}$$

Hence, the variation of ΔCS function in the face of an increase in transportation cost must be in the interval:

$$\left[\frac{5}{8}\sqrt{\frac{F}{t}}-\frac{5}{8}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}},\frac{5}{8}\sqrt{\frac{F}{t}}-\frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}}\right]$$

whenever $r^{**}>\frac{\theta}{3}$. For the case in which $r^{**}\leq\frac{\theta}{3}$ the partial derivative of ΔCS function is equal to the following:

$$\frac{\partial \Delta CS\left(r^{***},c,t,F,\theta\right)}{\partial t}=\frac{5}{8}\sqrt{\frac{F}{t}}-\frac{1}{2}\sqrt{\frac{F}{t}} \tag{64}$$

Case 3: The case of F is very similar to the one of t. It is enough to interchange t with F and vice versa. Hence:

$$\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial F}\in\left[\frac{5}{8}\sqrt{\frac{t}{F}}-\frac{5}{8}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{t}{F}},\frac{5}{8}\sqrt{\frac{t}{F}}-\frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{t}{F}}\right] \tag{65}$$

Otherwise, $\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial F}=\frac{5}{8}\sqrt{\frac{t}{F}}-\frac{1}{2}\sqrt{\frac{t}{F}}$ whenever $r^{**}\leq\frac{\theta}{3}$.

Case 4: Since $\frac{\partial \widehat{CS}(\cdot)}{\partial\theta}=0$, it is clear that whenever $r^{**}>\frac{\theta}{3}$, the following holds:

$$\frac{\partial \Delta CS\left(r^{***},c,t,F,\theta\right)}{\partial\theta}\in\left[0,\frac{1}{2}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{tF}\left(\frac{1}{r^{**}+\theta}-\frac{1}{4r^{**}}\right)\right] \tag{66}$$

For the case in which $r^{**}\leq\frac{\theta}{3}$, it is easy to show that:

$$\frac{\partial \Delta CS\left(r^{***},c,t,F,\theta\right)}{\partial\theta}=0 \tag{67}$$

This completes the proof.

Proof of Proposition 7

Proof. We know that at the optimal reference price r^{***} the difference in social welfare function satisfies the expression:

$$\Delta CS\left(r^{***}, c, t, F, \theta\right) = U - r^{***} - \tfrac{1}{4}\left(\tfrac{tF}{r^{***}-c}\right) - \left(U - c - \sqrt{tF} - \tfrac{1}{4}\sqrt{tF}\right) \tag{68}$$

We also know that the optimal reference price satisfies $r^{***} = r^{**}$ whenever $r^{**} > \frac{\theta}{3}$, otherwise $r^{***} = c + \frac{1}{2}\sqrt{tF}$. Hence, for the case when $r^{**} \leq \frac{\theta}{3}$ equation (68) reduces to $\Delta CS\left(r^{***}, c, t, F, \theta\right) = \frac{1}{4}\sqrt{tF} > 0$. When $r^{**} > \frac{\theta}{3}$, the optimal reference price satisfies $r^{***} = c + \sqrt{\frac{r^{***}}{r^{***}+\theta}}\sqrt{tF}$; hence, equation (68) reduces to $\Delta CS\left(r^{***}, c, t, F, \theta\right) = \left(\frac{5}{4} - \sqrt{\frac{r^{***}}{r^{***}+\theta}} - \frac{1}{4}\sqrt{\frac{r^{***}+\theta}{r^{***}}}\right)\sqrt{tF}$. It easy to show that the derivative of the function $f\left(r\right) = \sqrt{\frac{r}{r+\theta}} + \frac{1}{4}\sqrt{\frac{r+\theta}{r}}$ satisfies the following:

$$f'\left(r\right) = \frac{\theta}{2r}\sqrt{\frac{r}{r+\theta}}\left(\frac{1}{r+\theta} - \frac{1}{r}\right) \tag{69}$$

It is clear that $f'\left(r\right) > 0$ whenever $r^{**} > \frac{\theta}{3}$ and $\lim f\left(r\right) = \frac{5}{4}$ as $r \to \infty$, hence $\Delta CS\left(r^{***}, c, t, F, \theta\right) > 0$ whenever $r^{**} > \frac{\theta}{3}$. This completes the proof.