Hierarchical radiative quark mass matrices with an $\mathrm{U}(1)_{\mathrm{X}}$ horizontal symmetry model

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In a model with a gauge group $\mathrm{G}_{\mathrm{SM}} \otimes \mathrm{U}(1)_{\mathrm{X}}$, where $\mathrm{G}_{\mathrm{SM}} \equiv \mathrm{SU}(3)_{\mathrm{C}} \otimes \mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{U}(1)_{\mathrm{Y}}$ is the standard model gauge group and $\mathrm{U}(1)_{\mathrm{X}}$ is a horizontal local gauge symmetry, we propose a radiative generation of the spectrum of quark masses and mixing angles. The assignment of horizontal charges is such that at tree level only the third family is massive. Using these tree level masses and introducing exotic scalars, the light families of quarks acquire hierarchical masses through radiative corrections. The rank three quark mass matrices obtained are written in terms of a minimal set of free parameters of the model, whose values are estimated performing a numerical fit. The resulting quark masses and CKM mixing angles turn out to be in good agreement with the experimental values.

Keywords: Horizontal symmetry; quark masses; CKM

En un modelo con grupo de norma $\mathrm{G}_{\mathrm{SM}} \otimes \mathrm{U}(1)_{\mathrm{X}}$, donde $\mathrm{G}_{\mathrm{SM}} \equiv \mathrm{SU}(3)_{\mathrm{C}} \otimes \mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{U}(1)_{\mathrm{Y}}$ es el grupo de norma del modelo estándar y $\mathrm{U}(1)_{\mathrm{X}}$ es el grupo de una simetría horizontal local, se propone la generación radiativa del espectro de masas y los ángulos de mezcla de los quarks. La asignación de las cargas horizontales es tal que a nivel árbol solamente la tercera familia es masiva. Usando las masas a nivel árbol e introduciendo escalares exóticos, los quarks de las familias ligeras adquieren masas jerárquicas a través de correcciones radiativas. Las matrices de rango tres que se obtiene se expresan en términos de un conjunto mínimo de parámetros del modelo, cuyos valores se estiman realizando un ajuste numérico. Las masas de los quarks y los ángulos de mezcla obtenidos están en acuerdo con los valores experimentales.

Descriptores: Simetría horizontal; masas de los quarks; CKM

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1. Introduction

Although for several years a great effort has been done to shed some light on the mystery of the fermion masses it still is one of the outstanding puzzles of particle physics. There have been different approaches to explain the mass hierarchy, the fermion mixing and their possible relation to new physics. A good review covering widely these topics has been presented in Ref. 1. In this work we will restrict our study to that of a new horizontal symmetry and the derived radiative corrections.

A possible answer to why the masses of the light quarks are so small compared with the electroweak scale is that they arise from radiative corrections [2], while the mass of the top quark and possibly those of the bottom quark and of the tau lepton are generated at tree level. This may be understood as a consequence of the breaking of a symmetry among families (a horizontal symmetry). This symmetry may be discrete [3] or continuous [4]. Here we consider the case of a continuous local horizontal symmetry, a $\mathrm{U}(1)_{\mathrm{X}}$ gauge group broken spontaneously. We limit our calculation of masses to those of the quark sector, insisting that at tree level only the top and bottom quarks acquire mass. Instead of assuming a texture for the quark masses from the beginning, we carry out a one loop and a two loop calculation of the mass matrices in terms of some parameters which are the tree level top and bottom quark masses, one Yukawa coupling and the entries in the mass matrices of the scalar bosons that participate in the loop diagrams which contribute to the quark masses.

This paper is organized in the following way: In Sec. II we describe explicitly the model, Sec. III contains the analytical calculations, while Sec. IV is devoted to a numerical fit of our equations. The conclusions are presented in Sec. V.

2. The model

We assume only three families, the standard model (SM) families, and we do not introduce exotic fermions to cancel anomalies. The fermions are classified as in the SM in five sectors $f = q, u, d, l$ and e, where q and l are the $\mathrm{SU}(2)_{\mathrm{L}}$ quark and lepton doublets, respectively and u, d and e are the singlets, in an obvious notation. In order to reduce the number of parameters and to make the model free of anomalies, we demand that the values X of the horizontal charge satisfy the traceless condition [5]

$$X(f_i) = 0, \pm\delta_{\mathrm{f}}, \tag{1}$$

TABLE I. Horizontal charges of fermions.

Sector	Family 1	2	3
q	$\pm\Delta$	$\mp\Delta$	0
u	$\pm\delta$	$\mp\delta$	0
d	$\pm\delta$	$\mp\delta$	0
l	$\pm\Delta$	$\mp\Delta$	0
e	$\pm\delta$	$\mp\delta$	0

TABLE II. Quantum numbers for scalar fields, C denotes the dimension of the representation under the $\mathrm{SU}(3)_c$ color group.

	Class I		Class II					
	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7	ϕ_8
X	0	$-\delta$	0	Δ	0	δ	0	δ
Y	1	0	$-2/3$	$-2/3$	$4/3$	$4/3$	$-8/3$	$-8/3$
T	$1/2$	0	1	1	0	0	0	0
C	1	1	$\bar{6}$	$\bar{6}$	$\bar{6}$	$\bar{6}$	$\bar{6}$	$\bar{6}$

where $i = 1, 2, 3$ is a family index, with the constraint

$$\delta_{\rm q}^2 - 2\delta_{\rm u}^2 + \delta_{\rm d}^2 = \delta_{\rm l}^2 - \delta_{\rm e}^2. \tag{2}$$

Equation (1) guarantees the cancellation of the $[\mathrm{U}(1)_{\rm H}]^3$ anomaly as well as those which are linear in the $\mathrm{U}(1)_{\rm H}$ hypercharge ($[\mathrm{SU}(3)_{\rm C}]^2\mathrm{U}(1)_{\rm H}$, $[\mathrm{SU}(2)_{\rm L}]^2\mathrm{U}(1)_{\rm H}$, $[\mathrm{Grav}]^2\mathrm{U}(1)_{\rm H}$ and $[\mathrm{U}(1)_{\rm Y}]^2\mathrm{U}(1)_{\rm H}$). Equation (2) is the condition for the cancellation of the $\mathrm{U}(1)_{\rm Y}[\mathrm{U}(1)_{\rm H}]^2$ anomaly. A solution of Eq. (2) which guarantees that only the top and bottom quarks get masses at tree level is given by ("doublets independent of singlets", see Ref. 5),

$$\delta_{\rm l} = \delta_{\rm q} = \pm\Delta \neq \delta_{\rm u} = \delta_{\rm d} = \delta_{\rm e} = \pm\delta. \tag{3}$$

To avoid tree level flavor changing neutral currents, we do not allow mixing between the standard model Z boson and its horizontal counterpart. Consequently the SM Higgs scalar should have zero horizontal charge. As a consequence, and since we insist in having a non-zero tree-level mass for the top and bottom quarks, the horizontal charges of these quarks should satisfy

$$-X(q_3) + X(u_3) = 0,$$
$$-X(q_3) + X(d_3) = 0, \tag{4}$$

in order for the Yukawa couplings in Eq. (6) to be invariant, but then Eqs. (1) and (3) demand that they vanish,

$$X(u_3) = X(q_3) = X(d_3) = 0, \tag{5}$$

which in turn implies $X(l_3) = X(e_3) = 0$ (this defines the third family). The assignment of horizontal charges to the fermions is then as given in Table I. The $\mathrm{SU}(3)_{\rm C} \otimes \mathrm{SU}(2)_{\rm l} \otimes \mathrm{U}(1)_{\rm Y}$ quantum numbers of the fermions are the same as in the standard model.

To generate the first and second family quark masses radiatively we must introduce new irreducible representations (*irreps*) of scalar fields, since the gauge bosons of $\mathrm{G} = \mathrm{G}_{\rm SM} \otimes \mathrm{U}(1)_{\rm X}$ do not perform transitions between different families. Families are of course distinguishable (non-degenerated) only below the scale of the SM symmetry breaking, when they become massive.

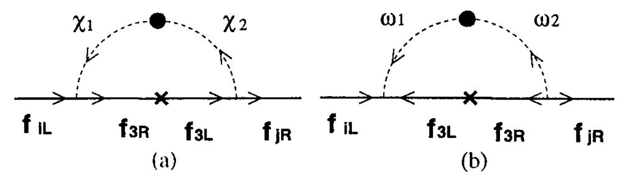

FIGURE 1. Generic diagrams that could contribute to the mass of the light families, a) D type couplings are represented with vertices where one fermion is incoming and the other one is outgoing and b) the M type couplings are represented with vertices where both fermions are incoming or outgoing.

Looking for scalars which make possible the generation of fermion masses in a hierarchical manner, we divide the *irreps* of scalar fields into two classes. Class I (II) contains scalar fields which get (do not get) vacuum expectation value (VEV).

A proper choice of scalars should be such that no VEVs are induced, through couplings in the potential, for scalars in class II. In the model considered below scalars of class II have no electrically neutral components, so they never get out of its class. In our model we introduce two *irreps* of scalars of class I and six *irreps* of scalars of class II, with the quantum numbers specified in Table II. Notice that we introduce just the minimum number of scalars of class I; that is, only one Higgs doublet of weak isospin to achieve the spontaneous symmetry breaking (SSB) of the electroweak group down to the electromagnetic $\mathrm{U}(1)_{\rm Q}$, and one $\mathrm{SU}(2)_{\rm L}$ singlet ϕ_2 used to break $\mathrm{U}(1)_{\rm X}$. In this way the horizontal interactions affect the ρ parameter only at higher orders.

With the above quantum numbers the quark Yukawa couplings that can be written may be divided into two classes, those of the D type which are defined by Fig. 1a, and those of the M type which are defined in Fig. 1b. The Yukawa couplings can thus be written as $L_{\rm Y} = L_{\rm Y_D} + L_{\rm Y_{\it M}}$, where the D Yukawa couplings are

$$L_{\rm Y_D} = Y^{\rm u}\bar{q}_{\rm L_3}\tilde{\phi}_1 u_{\rm R_3} + Y^{\rm d}\bar{q}_{\rm L_3}\phi_1 d_{\rm R_3} + \text{h.c.}, \tag{6}$$

with $\tilde{\phi} \equiv i\sigma_2\phi^*$, while the M couplings compatible with the symmetries of the model are

$$L_{\rm Y_M} = Y_I\Big[q_{\rm 1L}^{\alpha \rm T} C\phi_{3\{\alpha\beta\}}q_{\rm 2L}^{\beta} + q_{\rm 3L}^{\alpha \rm T} C\phi_{3\{\alpha\beta\}}q_{\rm 3L}^{\beta} + q_{\rm 2L}^{\alpha \rm T} C\phi_{4\{\alpha\beta\}}q_{\rm 3L}^{\beta} + d_{\rm 2R}^{\rm T} C\phi_5 d_{\rm 1R} + d_{\rm 3R}^{\rm T} C\phi_5 d_{\rm 3R} + d_{\rm 3R}^{\rm T} C\phi_6 d_{\rm 2R}$$
$$+ u_{\rm 2R}^{\rm T} C\phi_7 u_{\rm 1R} + u_{\rm 3R}^{\rm T} C\phi_7 u_{\rm 3R} + u_{\rm 3R}^{\rm T} C\phi_8 u_{2R}\Big] + \text{h.c.} \tag{7}$$

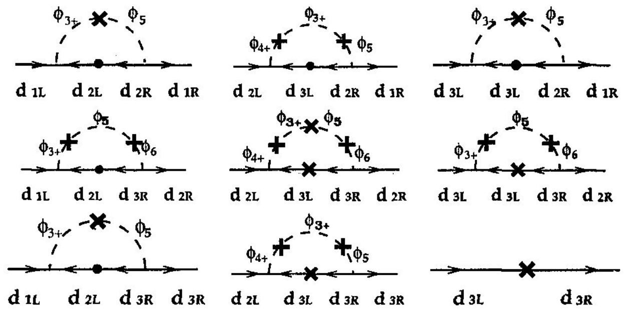

FIGURE 2. Mass matrix elements for d quarks.

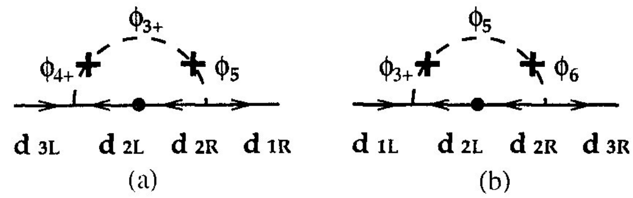

FIGURE 3. Extra diagrams that contributes to mass matrix elements, a) $(1,3)$ and b) $(3,1)$.

In these couplings C represents the charge conjugation matrix and α and β are weak isospin indices. Color indices have not been written explicitly. By simplicity and economy we have assumed only one Yukawa constant Y_I for all the M couplings. Notice that $\phi_{3\{\alpha\beta\}}$ is represented as

$$\phi_3 = \begin{pmatrix} \phi^{-4/3} & \phi^{-1/3} \\ \phi^{-1/3} & \phi^{2/3} \end{pmatrix} \tag{8}$$

where the superscript denotes the electric charge of the field. The same applies for ϕ_4.

Scalar fields which are not $\mathrm{SU}(2)_{\mathrm{L}}$ doublets do not participate in D type Yukawa terms, they however contribute to the mass matrix of the scalar sector and in turn determine the magnitude of the radiatively generated masses of fermions, as we shall see below.

The most general scalar potential of dimension ≤ 4 that can be written is

$$-V(\phi_i) = \sum_i \mu_i^2 |\phi_i|^2 + \sum_{i,j} \lambda_{ij} |\phi_i|^2 |\phi_j|^2 + \eta_{31}\phi_1^\dagger\phi_3^\dagger\phi_3\phi_1 + \tilde{\eta}_{31}\tilde{\phi}_1^\dagger\phi_3^\dagger\phi_3\tilde{\phi}_1 + \eta_{41}\phi_1^\dagger\phi_4^\dagger\phi_4\phi_1 + \tilde{\eta}_{41}\tilde{\phi}_1^\dagger\phi_4^\dagger\phi_4\tilde{\phi}_1 + \sum_{\substack{i\neq j \\ i,j\neq 1,2}} \eta_{ij} |\phi_i^\dagger \phi_j|^2$$
$$+ \rho_1 \phi_5^\dagger \phi_6 \phi_2 + \rho_2 \phi_7^\dagger \phi_8 \phi_2 + \lambda_1 \phi_5^\dagger \phi_1^\alpha \phi_{3\{\alpha\beta\}} \phi_1^\beta + \lambda_2 \phi_7^\dagger \tilde{\phi}_1^\alpha \phi_{3\{\alpha\beta\}} \tilde{\phi}_1^\beta + \lambda_3 \mathrm{Tr}(\phi_3^\dagger \phi_4) \phi_2^2 + \lambda_4 \phi_5 \phi_6 \phi_7 \phi_2 + \lambda_5 \phi_5 \phi_6^\dagger \phi_7^\dagger \phi_8 + \text{h.c.}, \tag{9}$$

where Tr means trace and in $|\phi_i|^2 \equiv \phi_i^\dagger \phi_i$ an appropriate contraction of the $\mathrm{SU}(2)_{\mathrm{L}}$ and $\mathrm{SU}(3)_{\mathrm{C}}$ indices is understood. The gauge invariance of this potential requires the relation $\Delta = 2\delta$.

Now we proceed to describe the mechanism that produces the quark masses. In general we could have contributions of two types as depicted in Fig. 1. In the present model however, we have only the diagrams of Fig. 2 for the charge $-1/3$ quark mass matrix elements and similar ones for the charge $2/3$ sector (these type of diagrams were first introduced in Ref. 2); in those diagrams of Fig. 2 the cross means tree level mixing and the black circle means one loop mixing. The diagrams in Fig. 3a and 3b should be added to the matrix elements $(1,3)$ and $(3,1)$, respectively. In the one loop contribution to the mass matrices for the different quark sectors only the third family of quarks appears in the internal lines. This generates a rank 2 matrix, which once diagonalized gives the physical states at this approximation. Then using these mass eigenstates we compute the next order contribution, obtaining a matrix of rank 3. After the diagonalization of this matrix we get the mass eigenvalues and eigenstates (A quark mass mechanism with some similar features to the one proposed here is given in Ref. 2).

The VEVs of the class I scalar fields are

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle \phi_2 \rangle = v_2, \tag{10}$$

and they achieve the breaking

$$\mathrm{G}_{\mathrm{SM}} \otimes \mathrm{U}(1)_{\mathrm{X}} \xrightarrow{\langle \phi_2 \rangle} \mathrm{G}_{\mathrm{SM}} \xrightarrow{\langle \phi_1 \rangle} \mathrm{SU}(3)_{\mathrm{C}} \otimes \mathrm{U}(1)_{\mathrm{Q}}. \tag{11}$$

The scalar field mixing arises after SSB from the terms in the potential that couple two different class II fields to one of class I. After SSB the mass matrices for the scalar fields of charge $2/3(\phi_4, \phi_3, \phi_5, \phi_6)$ and $4/3(\phi_4, \phi_3, \phi_7, \phi_8)$ are written, respectively, as

$$M_{2/3}^2 = \begin{pmatrix} s_4^2 & \lambda_3^* v_2^2 & 0 & 0 \\ \lambda_3 v_2^2 & s_3^2 & \dfrac{\lambda_1^* v_1^2}{2} & 0 \\ 0 & \dfrac{\lambda_1 v_1^2}{2} & u_5^2 & \rho_1 v_2 \\ 0 & 0 & \rho_1^* v_2 & u_6^2 \end{pmatrix},$$

$$M_{4/3}^2 = \begin{pmatrix} t_4^2 & \lambda_3^* v_2^2 & 0 & 0 \\ \lambda_3 v_2^2 & t_3^2 & \dfrac{\lambda_2 v_1^2}{2} & 0 \\ 0 & \dfrac{\lambda_2^* v_1^2}{2} & t_7^2 & \rho_2 v_2 \\ 0 & 0 & \rho_2^* v_2 & t_8^2 \end{pmatrix}, \tag{12}$$

where from Eq. (9) $t_i^2 = u_i^2 = \mu_i^2 + \lambda_{i1} v_1^2 + \lambda_{i2} v_2^2$ and $s_i^2 = t_i^2 + \eta_{i1} v_1^2$.

Notice that due to the scalar mixing in all the loop diagrams of Fig. 2 and 3, the divergences in each one of these diagrams cancel as is physically expected, giving rise to finite contributions to the quark mass matrices.

Explicitly, the non vanishing contributions from the diagrams of Fig. 2 to the mass terms $\bar{d}_{i\mathrm{R}} d_{j\mathrm{L}} \Sigma_{ij}^{(1)} + \mathrm{h.c.}$ read at one loop

$$\Sigma_{22}^{(1)} = 3m_{\mathrm{b}}^{(0)} \frac{Y_I^2}{16\pi^2} \sum_k \mathrm{U}_{1k}\mathrm{U}_{4k} \mathrm{f}\big(M_k, m_{\mathrm{b}}^{(0)}\big), \tag{13}$$

$$\Sigma_{23}^{(1)} = 3m_{\mathrm{b}}^{(0)} \frac{Y_I^2}{16\pi^2} \sum_k \mathrm{U}_{2k}\mathrm{U}_{4k} \mathrm{f}\big(M_k, m_{\mathrm{b}}^{(0)}\big), \tag{14}$$

$$\Sigma_{32}^{(1)} = 3m_{\mathrm{b}}^{(0)} \frac{Y_I^2}{16\pi^2} \sum_k \mathrm{U}_{1k}\mathrm{U}_{3k} \mathrm{f}\big(M_k, m_{\mathrm{b}}^{(0)}\big), \tag{15}$$

where $m_{\mathrm{b}}^{(0)}$ is the tree level contribution to the b quark mass, the 3 is a color factor, U is the orthogonal matrix which diagonalizes the mass matrix of the charge $2/3$ scalars,

$$\big(\phi_4, \phi_3, \phi_5, \phi_6\big)^{\mathrm{T}} = \mathrm{U}(\sigma_1, \sigma_2, \sigma_3, \sigma_4)^{\mathrm{T}},$$

where σ_i are the eigenfields with eigenvalues M_i, and

$$f(a,b) \equiv \frac{1}{a^2 - b^2} a^2 \ln \frac{a^2}{b^2},$$

which is just a logarithmic contribution when $a^2 \gg b^2$. The resulting second rank mass matrix at this level is thus

$$M_{\mathrm{d}}^{(1)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Sigma_{22}^{(1)} & \Sigma_{23}^{(1)} \\ 0 & \Sigma_{32}^{(1)} & m_{\mathrm{b}}^{(0)} \end{pmatrix}. \tag{16}$$

At effective two loops we obtain the following expressions:

$$\Sigma_{11}^{(2)} = 3\frac{Y_I^2}{16\pi^2} \sum_{k,i} m_i^{(1)} \big(V_{\mathrm{dL}}^{(1)}\big)_{2i} \big(V_{\mathrm{dR}}^{(1)}\big)_{2i} \mathrm{U}_{2k}\mathrm{U}_{3k} \mathrm{f}\big(M_k, m_i^{(1)}\big), \tag{17}$$

$$\Sigma_{12}^{(2)} = 3\frac{Y_I^2}{16\pi^2} \sum_{k,i} m_i^{(1)} \big(V_{\mathrm{dL}}^{(1)}\big)_{3i} \big(V_{\mathrm{dR}}^{(1)}\big)_{2i} \mathrm{U}_{1k}\mathrm{U}_{3k} \mathrm{f}\big(M_k, m_i^{(1)}\big), \tag{18}$$

$$\Sigma_{13}^{(2)} = 3\frac{Y_I^2}{16\pi^2} \sum_{k,i} m_i^{(1)} \big(V_{\mathrm{dL}}^{(1)}\big)_{3i} \big(V_{\mathrm{dR}}^{(1)}\big)_{2i} \mathrm{U}_{2k}\mathrm{U}_{3k} \mathrm{f}\big(M_k, m_i^{(1)}\big) + 3\frac{Y_I^2}{16\pi^2} \sum_{k,i} m_i^{(1)} \big(V_{\mathrm{dL}}^{(1)}\big)_{2i} \big(V_{\mathrm{dR}}^{(1)}\big)_{2i} \mathrm{U}_{1k}\mathrm{U}_{3k} \mathrm{f}\big(M_k, m_i^{(1)}\big), \tag{19}$$

$$\Sigma_{21}^{(2)} = 3\frac{Y_I^2}{16\pi^2} \sum_{k,i} m_i^{(1)} \big(V_{\mathrm{dL}}^{(1)}\big)_{2i} \big(V_{\mathrm{dR}}^{(1)}\big)_{3i} \mathrm{U}_{2k}\mathrm{U}_{4k} \mathrm{f}\big(M_k, m_i^{(1)}\big), \tag{20}$$

$$\Sigma_{31}^{(2)} = 3\frac{Y_I^2}{16\pi^2} \sum_{k,i} m_i^{(1)} \big(V_{\mathrm{dL}}^{(1)}\big)_{2i} \big(V_{\mathrm{dR}}^{(1)}\big)_{3i} \mathrm{U}_{2k}\mathrm{U}_{3k} \mathrm{f}\big(M_k, m_i^{(1)}\big) + 3\frac{Y_I^2}{16\pi^2} \sum_{k,i} m_i^{(1)} \big(V_{\mathrm{dL}}^{(1)}\big)_{2i} \big(V_{\mathrm{dR}}^{(1)}\big)_{2i} \mathrm{U}_{2k}\mathrm{U}_{4k} \mathrm{f}\big(M_k, m_i^{(1)}\big), \tag{21}$$

where the k (i) index goes from 1 to 4 (from 2 to 3), $V_{\mathrm{dL}}^{(1)}$ and $V_{\mathrm{dR}}^{(1)}$ are the unitary matrices which diagonalize $M_{\mathrm{d}}^{(1)}$ of Eq. (16) and $m_i^{(1)}$ are the eigenvalues. Therefore at two loops the mass matrix for d quarks becomes

$$M_{\mathrm{d}}^{(2)} = \begin{pmatrix} \Sigma_{11}^{(2)} & \Sigma_{12}^{(2)} & \Sigma_{13}^{(2)} \\ \Sigma_{21}^{(2)} & m_2^{(1)} & 0 \\ \Sigma_{31}^{(2)} & 0 & m_3^{(1)} \end{pmatrix}. \tag{22}$$

For the up sector the procedure to obtain the masses is completely analogous. That is, the mass terms for the up sector come from graphs like those in Figs. 2 and 3, but replacing the ϕ_4, ϕ_3, ϕ_5 and ϕ_6 scalar fields by ϕ_4, ϕ_3, ϕ_7 and ϕ_8 and the quarks d_i by the quarks u_i.

The CKM matrix takes the form

$$V_{\mathrm{CKM}} = \Big(V_{\mathrm{uL}}^{(2)} V_{\mathrm{uL}}^{(1)}\Big)^{\dagger} V_{\mathrm{dL}}^{(2)} V_{\mathrm{dL}}^{(1)}, \tag{23}$$

where the unitary matrices $V_{\mathrm{uL}}^{(1)}$ and $V_{\mathrm{uR}}^{(1)}$ diagonalize $M_{\mathrm{u}}^{(1)}$ and $V_{\mathrm{uL}}^{(2)}$ and $V_{\mathrm{uR}}^{(2)}$ diagonalize $M_{\mathrm{u}}^{(2)}$, with an analogous notation used for the down sector.

It is important to mention here that the textures, particularly the zeros in the scalar and quark mass matrices [Eqs. (12) and (22)] are not accidental neither imposed; they are just a direct consequence of the mass mechanism that we are introducing and of the gauge symmetry of the model.

3. Numerical evaluation

3.1. Experimental values

Since quarks are confined inside hadrons, their masses can not be directly measured. So, the quark mass parameters in the SM lagrangian depend both on the renormalization scheme adopted to define the theory and on the scale parameter μ where the theory is being tested. In the limit where all quark masses are zero, the SM has an $\mathrm{SU}(3)_{\mathrm{L}} \otimes \mathrm{SU}(3)_{\mathrm{R}}$ chiral symmetry which is broken at an scale $\Lambda_\chi \simeq 1$ GeV. To determine the quark mass values one must use SM perturbation theory at an energy scale $\mu \gg \Lambda_\chi$ where non perturbative effects are negligible. For illustration, the allowed ranges of quark masses [6] in the modified minimal subtraction scheme $(\overline{MS})$ are [7]:

$$\begin{aligned} m_{\mathrm{u}}(1\ \mathrm{GeV}) &= 2 - 6.8\ \mathrm{MeV}, \\ m_{\mathrm{d}}(1\ \mathrm{GeV}) &= 4 - 12\ \mathrm{MeV}, \\ m_{\mathrm{s}}(1\ \mathrm{GeV}) &= 81 - 230\ \mathrm{MeV}, \\ m_{\mathrm{c}}(m_{\mathrm{c}}) &= 1.1 - 1.4\ \mathrm{GeV}, \\ m_{\mathrm{b}}(m_{\mathrm{b}}) &= 4.1 - 4.4\ \mathrm{GeV}, \\ m_{\mathrm{t}}(\mathrm{Exp}) &= 173.8 \pm 5.2\ \mathrm{GeV}. \end{aligned}$$

TABLE III. Experimentally allowed values for $m_{\mathrm{q}}(m_{\mathrm{t}})$ and CKM matrix elements. We show the input and calculated values in the context of our model.

	Range	Input	Best fit
$m_{\mathrm{d}}(m_{\mathrm{t}})$	3.85–5.07 MeV	4.46 MeV	3.63 MeV
$m_{\mathrm{s}}(m_{\mathrm{t}})$	76.9–100 MeV	88.4 MeV	44.9 MeV
$m_{\mathrm{b}}(m_{\mathrm{t}})$	2.74–2.96 GeV	2.85 GeV	2.91 GeV
$m_{\mathrm{u}}(m_{\mathrm{t}})$	1.8–2.63 MeV	2.21 MeV	2.22 MeV
$m_{\mathrm{c}}(m_{\mathrm{t}})$	587–700 MeV	643 MeV	841 MeV
$m_{\mathrm{t}}(m_{\mathrm{t}})$	159–183 GeV	171 GeV	166.5 GeV
CKM_{11}	0.9745–0.9760	0.9752	0.9761
CKM_{12}	0.217–0.224	0.2200	0.2179
CKM_{13}	0.0018–0.0045	0.0034	0.0032
CKM_{21}	0.217–0.224	0.2200	0.2214
CKM_{22}	0.9737–0.9753	0.9755	0.9742
CKM_{23}	0.036–0.042	0.0390	0.0382
CKM_{31}	0.004–0.013	0.0085	0.0117
CKM_{32}	0.035–0.042	0.0385	0.0365
CKM_{33}	0.9991–0.9994	0.9992	0.9992

TABLE IV. Experimentally allowed values for $m_{\mathrm{q}}(M_Z)$ and CKM matrix elements. We show the input and calculated values in the context of our model.

	Range	Input	Best fit
$m_{\mathrm{d}}(M_Z)$	1.8–5.3 MeV	3.55 MeV	3.44 MeV
$m_{\mathrm{s}}(M_Z)$	35–100 MeV	67.5 MeV	39.6 MeV
$m_{\mathrm{b}}(M_Z)$	2.8–3.0 GeV	2.9 GeV	2.9 GeV
$m_{\mathrm{u}}(M_Z)$	0.9–2.9 MeV	1.19 MeV	2.03 MeV
$m_{\mathrm{c}}(M_Z)$	530–680 MeV	605 MeV	793 MeV
$m_{\mathrm{t}}(M_Z)$	168–180 GeV	174 GeV	166.9 GeV
CKM_{11}	0.9745–0.9760	0.9752	0.9762
CKM_{12}	0.217–0.224	0.2205	0.2174
CKM_{13}	0.0018–0.0045	0.0036	0.0030
CKM_{21}	0.217–0.224	0.2205	0.2215
CKM_{22}	0.9737–0.9753	0.9745	0.9741
CKM_{23}	0.036–0.042	0.0390	0.0387
CKM_{31}	0.004–0.013	0.0085	0.0116
CKM_{32}	0.035–0.042	0.0385	0.0370
CKM_{33}	0.9991–0.9994	0.9992	0.9992

To get the relative magnitude of different quark masses in a meaningful way, one has to describe all quark masses in the same scheme and at the same scale. In our analysis we are calculating the quark masses at an energy scale μ_m such that $M_Z < \mu_m < M_{\mathrm{X}} \simeq v_2$, where M_{X} is the mass scale where $\mathrm{U}(1)_{\mathrm{X}}$ is spontaneously broken. Since in our model there is no mixing between the standard model Z boson and its horizontal counterpart, we can have v_2 as low as the electroweak breaking scale. For simplicity, let us assume that our calculations are meaningful at the electroweak breaking scale and from the former values for the quark masses let us calculate, in the $\overline{MS}$ scheme, the quark masses at the m_{t} scale [8] and at the M_Z scale [1]. Those values calculated in the references cited, are presented in Tables III and IV, respectively. On the other hand, the CKM matrix elements are not ill defined and they can be directly measured from the charged weak current in the SM. For simplicity we assume that they are real, and as discussed in Ref. 8, they are almost constant in the interval $M_Z < \mu <$ a few TeV. Their current experimental value [6] are given in the Tables III and IV.

3.2. Evaluation of the parameters

In order to test the model using the least possible number of free parameters, let us write the scalar mass matrices in the following form:

$$M_{2/3}^2 = \begin{pmatrix} a_+ & b & 0 & 0 \\ b & a_+ & c_+ & 0 \\ 0 & c_+ & a_+ & d_+ \\ 0 & 0 & d_+ & a_+ \end{pmatrix},$$

$$M_{4/3}^2 = \begin{pmatrix} a_- & b & 0 & 0 \\ b & a_- & c_- & 0 \\ 0 & c_- & a_- & d_- \\ 0 & 0 & d_- & a_- \end{pmatrix}. \tag{24}$$

Using the central value of the CKM elements in the PDG book [6] and the central values of the six quark masses at the top mass scale [8], we build the χ^2 function in the ten parameter space defined by $\left[a_+, a_-, b, c_+, c_-, d_+, d_-, Y_I, m_{\mathrm{b}}^{(0)}, m_{\mathrm{t}}^{(0)}\right]$, where $m_{\mathrm{b}}^{(0)}$ and $m_{\mathrm{t}}^{(0)}$ are the tree level quark masses for the bottom and top quarks respectively. Expressions for the eigenvectors and eigenvalues of the mass matrices involved in the numerical evaluation were obtained using MATHEMATICA, and the χ^2 function was minimized using MINUIT from the CERNLIB packages [9]; both Monte Carlo and standard routines were used in the minimization process. The tree level masses of the top $\left[m_{\mathrm{t}}^{(0)}\right]$ and bottom $\left[m_{\mathrm{b}}^{(0)}\right]$ quarks were restricted to be around the central values $\pm 10\%$ in order to assure consistency with the assumption that radiative corrections are small. The χ^2 function presents an even symmetry with respect to 5 of the parameters of the matrices in Eq. (24); we find that there are 32 parameter domains where the χ^2 function takes small values. For the extremal points this even symmetry is not an exact one, but all the zones have a Yukawa constant of the order of 10 and give masses and CKM matrix elements in good agreement with the available experimental values. The numerical results on one of the 32 minima of the ten parameter space are shown in Table V. We use those values (which minimize χ^2) to calculate, in the context of our model, the fifteen predictions for $m_{\mathrm{q}}(m_{\mathrm{t}})$ for $q = u, d, c, s, t, b$ and $(CKM)_{ij}$ for $i, j = 1, 2, 3$. The numerical results are shown in Table III. For the sake of comparison, we repeat the same calculations but now using the central values of the six quark masses at the M_Z scale [1]. The numerical results are shown in Table IV.

Let us make two comments: first, the values for the parameters in the scalar field square mass matrices are of order 10^{17} (MeV)^2 (see Table V), so, the scalar physical masses

TABLE V. Values of the parameters in one of the minima [the values of the scalar mass matrices elements are in $(\mathrm{MeV})^2$ units].

Parameter	Value	Parameter	Value
a_-	22.845×10^{16}	d_+	13.762×10^{16}
b	1.850×10^{16}	a_+	150.01×10^{16}
c_-	10.018×10^{16}	Y	13.6
d_-	-10.227×10^{16}	$m_{\mathrm{t}}^{(0)}$	2.912 GeV
c_+	13.571×10^{16}	$m_{\mathrm{b}}^{(0)}$	166.1 GeV

are of order 10^3 TeV. Second, the rounding errors allow us to take safely up to five significative figures in the masses and in the CKM matrix elements. As can be seen from Tables III and IV even, under the assumption that the CKM matrix elements are real, the numerical values are in good agreement with the allowed experimental results.

4. Conclusions

By introducing a $\mathrm{U}(1)_{\mathrm{X}}$ gauge flavor symmetry and enlarging the scalar sector, we have presented a mechanism and an explicit model able to generate radiatively the hierarchical spectrum of quarks masses and CKM mixing angles. The horizontal charge assignment to particles is such that we do not need to go beyond the known three generations of quarks and leptons. Also, at tree level only the t and b quarks get masses. To generate radiatively the masses for the light families we have introduced some new exotic scalars. All of these new scalars are charged and color non-singlets, so they can not get VEV as is required in the loop graphs.

Our numerical results are presented in Tables III and IV. Even though we are guessing the $\mathrm{U}(1)_{\mathrm{X}}$ mass scale, the two sets of results do not differ by much and they agree fairly well with the experimental values, meaning that the mass scale associated with the horizontal symmetry may be in the range $100\ \mathrm{GeV} < M_{\mathrm{X}} < 1.0$ TeV. A closer look to our analysis shows that we are translating the quark mass hierarchy to the quotient v_1/v_2 which is the hierarchy between the electroweak mass scale and the horizontal $\mathrm{U}(1)_{\mathrm{X}}$ mass scale. In this way we demonstrate the viability that new physics at the electroweak mass scale, or just above it, may help to explain the long-lasting puzzle of the enormous range of quark masses and mixing angles.

Since quarks carry baryon number $B = 1/3$, the color sextet scalars we have introduced must have $B = -2/3$ (the scalar singlets ϕ_1 and ϕ_2 have $B = 0$); in this way $\mathrm{L}_{\mathrm{Y_M}}$ is not only $\mathrm{U}(1)_{\mathrm{X}}$ invariant but conserves color and baryon number as well. On the other hand, $V(\phi)$ does not conserve baryon number; as a matter of fact, the term $\lambda_4\phi_5\phi_6\phi_7\phi_2$ violates baryon number by two units ($\Delta B = \pm 2$) and could induce neutron-antineutron oscillations. A roughly estimate of such oscillations shows that they are proportional to $v_2(M^2_{\phi_5}M^2_{\phi_6}M^2_{\phi_7})^{-1} \sim 10^{-19}$ GeV which is negligible in principle. Any way, in the worse of the situations, since the offending term does not enter on the mass matrix for the Higgs scalars (it is just there), it may be removed in more realistic models by the introduction of a discrete symmetry.

Our results are encouraging; even under the assumption that the CKM matrix is real, and without knowing exactly the $\mathrm{U}(1)_{\mathrm{X}}$ mass scale, the numerical predictions are in the ballpark, implying also a value of order 10 for the Yukawa coupling Y_I, and masses for the exotic scalars being of order 10^3 TeV. Our model presents thus a clear mechanism able to explain the mass hierarchy and mixing of the quarks.

Finally let us mention that in the work presented here, the Higgs scalar used to produce the SSB of the SM gauge group down to $\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{U}(1)_{\mathrm{Q}}$ has zero horizontal charge, and as a consequence the standard Z boson does not mix with the horizontal counterpart. However, due to recent interest [10] on the phenomenology of a Z', it is worth to study the possibility of allowing this mixing in future work.

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