Polarization in the non-leptonic weak decays of spin-3/2 hyperons

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Recibido el 23 de octubre de 2001; aceptado el 4 de marzo de 2002

We compute the transition rate for polarized states in the decay of a spin-3/2 particle hyperon into a spin-$1/2$ baryon and a spin-0 meson.

Keywords: Polarized states; hyperons; transition rate.

Calculamos la razón de transición para estados polarizados en los decaimientos de hiperones de espín-3/2 en un barión de espín-$1/2$ y un mesón de espín-0.

Descriptores: Estados polarizados; hiperones; razón de transición.

PACS: 14.20.Jn; 13.88.+e

1. Introduction

Polarization of particles plays an important role in understanding particle interactions, in the study of angular distributions of the decay products when a polarized particle disintegrates, and in measurements of the electromagnetic moments. Polarization of a spin-1/2 particle is described by a vector, which is related to the spin of the particle. Polarization of a spin-3/2 particle is described by a vector, and quadrupole and octupole tensors, and so on.

For a spin-1/2 baryon (4-momentum p_1) decaying into another spin-1/2 baryon (4-momentum p_2), and a spin-0 meson (4-momentum k), a relation between the polarization vector of the parent baryon and that of the daughter is well known [1], and the transition rate is given by [2]

$$R = 1 + \gamma\omega_2\cdot\omega_1 + (1-\gamma)\omega_2\cdot\mathbf{n}\,\omega_1\cdot\mathbf{n} + \alpha(\omega_2\cdot\mathbf{n} + \omega_1\cdot\mathbf{n}) + \beta(\omega_2\times\omega_1)\cdot\mathbf{n}, \quad (1)$$

where

$$\alpha = \frac{2\Re(sp^*)}{|s|^2+|p|^2}, \quad \text{(2a)}$$

$$\beta = \frac{2\Im(sp^*)}{|s|^2+|p|^2}, \quad \text{(2b)}$$

and

$$\gamma = \frac{|s|^2-|p|^2}{|s|^2+|p|^2}. \quad \text{(2c)}$$

In Eq. (1), ω_1 (ω_2) is a unit vector in the direction of initial (final) baryon spin. The vector $\mathbf{n} = \mathbf{p}_2/|\mathbf{p}_2|$, $s = A$ and $p = |\mathbf{p}_2|\,B/(E_2+M_2)$, with A and B constants related to the scalar and pseudoscalar couplings. The hyperon magnetic dipole moment may be determined by allowing polarized hyperons to undergo Larmor spin precession in a known magnetic field and detecting the change in decay asymmetry. This results following Eq. (1) after summing over final baryon polarization.

The purpose of this short note is to present the analogous result to Eq. (1), for the decay of a spin-3/2 hyperon into a spin-1/2 baryon and a spin-0 meson, which as a typical example is the decay $\Omega^- \to \Xi^0\pi^-$. Ten years ago the Ω^- magnetic dipole moment was measured [3]. Our work complements the result in Ref. 4, where the analogous result of Ref. 1, *i.e.*, the relation between the spins of the parent spin-3/2 and the daughter spin-1/2 hyperons is reported.

In Sec. 2, we review the Rarita-Schwinger equation and some of the properties of the spin-3/2 particle. There we calculate the analogous formula to Eq. (1). In Sec. 3, we consider two possible experimental situations for the polarized spin-3/2 particle, and present our conclusions.

2. Spin-3/2 particle

A spin-3/2 particle can be described, *a lá* Rarita-Schwinger [5], by a vector-spinor wave function ψ_μ, constructed by the combination of a Dirac spinor (spin-1/2) and a Proca vector (spin-1). In momentum space we can write for this vector-spinor [6]

$$u_\mu(p,\lambda) = \sum_{\alpha\beta}\left\langle 1\frac{1}{2},\alpha\beta\left|\frac{3}{2}\lambda\right.\right\rangle\varepsilon_\mu(p,\alpha)u(p,\beta), \quad (3)$$

where λ,α and β are the helicities of the vector-spinor $u_\mu(p,\lambda)$, vector $\varepsilon_\mu(p,\alpha)$ and spinor $u(p,\beta)$, respectively. The coefficients in Eq. (3) are the Clebsh-Gordan symbols. Rarita-Schwinger equation must be satisfied

$$(\gamma^\alpha p_\alpha - m)u_\mu(p,\lambda) = 0, \quad (4)$$

along with the restriction equations

$$\gamma^\mu u_\mu(p,\lambda) = 0,$$
$$p^\mu u_\mu(p,\lambda) = 0, \quad (5)$$

which removes the unwanted spin-1/2 components, arising from the combination $\mathbf{1}\otimes\frac{1}{2}=\frac{3}{2}\oplus\frac{1}{2}$.

The spin-energy projection operator for spin-3/2 is given by [7]:

$$\sum(p)^{\mu\nu} = u^{\mu}(p,\kappa)u^{\nu}(p,\kappa) = \frac{m+\gamma^{\alpha}p_{\alpha}}{24m}\left\{-3g^{\mu\nu}+\gamma^{\mu}\gamma^{\nu}+\frac{2p^{\mu}p^{\nu}}{m^2}+\frac{p^{\mu}\gamma^{\nu}-p^{\nu}\gamma^{\mu}}{m}\right.$$
$$+a_1\left[-\frac{i}{m}\epsilon^{\mu\nu\lambda\tau}p_{\lambda}\eta_{\tau}+\frac{4}{5}I^{\mu\nu}\gamma_5\gamma^{\tau}\eta_{\tau}-\frac{1}{5}\gamma_5\gamma_{\rho}(I^{\rho\mu}\eta^{\nu}+I^{\rho\nu}\eta^{\mu})\right]$$
$$\left.-a_2\left[\eta^{\mu\nu}-\gamma_{\rho}(\eta^{\rho\mu}\gamma^{\nu}-\eta^{\rho\nu}\gamma^{\mu})-\frac{1}{m}\gamma_{\rho}(\eta^{\rho\mu}p^{\nu}-\eta^{\rho\nu}p^{\mu})\right]-a_3\gamma_5\gamma_{\rho}\eta^{\mu\nu\rho}\right\}, \quad (6)$$

with

$$a_1 = \frac{5(\kappa_1+\kappa_2+\kappa_3)+3\kappa_1\kappa_2\kappa_3}{4\,|\kappa|},$$
$$a_2 = \frac{\kappa_1\kappa_2+\kappa_2\kappa_3+\kappa_3\kappa_1}{2\,|\kappa|},$$
$$a_3 = \frac{\kappa_1\kappa_2\kappa_3}{|\kappa|},$$

where

$$\kappa_1=\kappa_2=\kappa_3=\pm1$$

for $\kappa=\pm\frac{3}{2}$,

$$\kappa_1=\kappa_2=-\kappa_3=\pm1$$

for $\kappa=\pm\frac{1}{2}$,

$$\eta^{\mu\nu} = 3\eta^{\mu}\eta^{\nu}-I^{\mu\nu},$$
$$\eta^{\mu\nu\rho} = \frac{9}{2}\eta^{\mu}\eta^{\nu}\eta^{\rho}-\frac{9}{10}\left(I^{\rho\mu}\eta^{\nu}+I^{\rho\nu}\eta^{\mu}+I^{\mu\nu}\eta^{\rho}\right),$$
$$I^{\mu\nu} = -g^{\mu\nu}+\frac{p^{\mu}p^{\nu}}{m^2},$$

and η^{μ} is the spin 4-vector. We also need the spin-energy projection operator for spin-1/2 [7]

$$u(p,\kappa)\overline{u}(p,\kappa) = \frac{m+\gamma^{\alpha}p_{\alpha}}{4m}\left[1+\frac{\kappa}{|\kappa|}\gamma_5\gamma_{\mu}\eta^{\mu}\right]. \quad (7)$$

With this at hand, we can now compute the polarization transition decay rate for the process spin-3/2→spin-1/2+spin-0. We start with the amplitude [8]

$$\mathcal{M} = \overline{u}(p_2)(A-B\gamma_5)k_{\mu}u^{\mu}(p_1). \quad (8)$$

The couplings A and B are parity conserving and parity violating, respectively. Next, we calculate

$$|\mathcal{M}|^2 = \overline{u}_{\mu}(p_1)(A^*+B^*\gamma_5)k^{\mu}u(p_2)$$
$$\times\overline{u}(p_2)(A-B\gamma_5)k_{\nu}u^{\nu}(p_1),$$

which can be rewritten as

$$|\mathcal{M}|^2 = k^{\mu}k^{\nu}\left[u_{\nu}(p_1)\right]_{\rho}\left[\overline{u}_{\mu}(p_1)\right]_{\alpha}\left[u(p_2)\right]_{\beta}\left[\overline{u}(p_2)\right]_{\sigma}$$
$$\times(A^*+B^*\gamma_5)_{\alpha\beta}(A-B\gamma_5)_{\sigma\rho}. \quad (9)$$

Replacing Eqs. (6) and (7) into Eq. (9) we obtain

$$|\mathcal{M}|^2 = k^{\mu}k^{\nu}\left[\sum(p_1)_{\nu\mu}\right]_{\rho\alpha}\left[\frac{\gamma^{\tau}p_{2\tau}+m_2}{2m_2}\right.$$
$$\left.\times\frac{1+\gamma_5\gamma^{\tau}s_{2\tau}}{2}\right]_{\beta\sigma}(A^*+B^*\gamma_5)_{\alpha\beta}(A-B\gamma_5)_{\sigma\rho}$$
$$= k^{\mu}k^{\nu}Tr\left[\sum(p_1)_{\nu\mu}(A^*+B^*\gamma_5)\frac{\gamma^{\tau}p_{2\tau}+m_2}{2m_2}\right.$$
$$\left.\times\frac{1+\gamma_5\gamma^{\tau}s_{2\tau}}{2}(A-B\gamma_5)\right]. \quad (10)$$

This, Eq. (10), is the expression to be simplified; we will do this in reference system of the decaying particle. Our definitions are

$$p_1^{\mu} = (m_1,0),$$
$$p_2^{\mu} = (E_2,\mathbf{p}_2),$$
$$k^{\mu} = (\omega,\overrightarrow{k}) = (m_1-E_2,-\mathbf{p}_2),$$
$$s_1^{\mu} = (0,\omega_1),$$
$$s_2^{\mu} = \left(\frac{\mathbf{p}_2\cdot\omega_2}{m_2},\omega_2+\frac{\mathbf{p}_2\cdot\omega_2}{m_2(E_2+m_2)}\mathbf{p}_2\right),$$

where, again, $\omega_1(\omega_2)$ is a unit vector in the direction of initial (final) baryon spin, and $\mathbf{n}=\mathbf{p}_2/|\mathbf{p}_2|$. Performing Dirac algebra in Eq. (10) we find that

$$|\mathcal{M}|^2 = \frac{E_2+m_2}{m_2}\,|\mathbf{p}_2|\left\{|p|^2+|d|^2+\frac{4}{5}\left(|p|^2-|d|^2\right)\omega_2\cdot\omega_1-\left(|p|^2+|d|^2\right)(\omega_1\cdot\mathbf{n})^2\right.$$
$$-\left(|p|^2-|d|^2\right)(\omega_1\cdot\mathbf{n})^2\,\omega_2\cdot\omega_1+\frac{E_2+m_2}{m_2}\,|d|^2\left[\frac{7}{5}(\omega_2\cdot\mathbf{n})(\omega_1\cdot\mathbf{n})-(\omega_1\cdot\mathbf{n})^3\,\omega_2\cdot\mathbf{n}\right]$$
$$\left.+2\Re(pd^*)\left[\frac{7}{5}(\omega_1\cdot\mathbf{n})+\omega_2\cdot\mathbf{n}-(\omega_1\cdot\mathbf{n})^2\left(\omega_2\cdot\mathbf{n}+\omega_1\cdot\mathbf{n}\right)\right]-2\Im(pd^*)\left[\frac{4}{5}-(\omega_1\cdot\mathbf{n})^2\right]\mathbf{n}\cdot(\omega_2\times\omega_1)\right\}. \quad (11)$$

We have oriented the spin of decaying particle along positive z axis, and defined

$$p = A,$$
$$d = \frac{B}{E_2+m_2}\,|\mathbf{p}_2|\,,$$

which are the $p-$wave and $d-$wave contributions, respectively.

To get the transition rate we divide Eq. (11) by the corresponding expression summed over spins,

$$\overline{|\mathcal{M}|^2} = \frac{E_2+m_2}{m_2}\,|\overrightarrow{p_2}|\left(|p|^2+|d|^2\right). \quad (12)$$

Then, the transition rate is given by

$$R = 1+\frac{2}{5}\gamma\mathbf{P}\cdot\omega_2+\alpha\left(\frac{7}{10}\mathbf{P}\cdot\mathbf{n}+\omega_2\cdot\mathbf{n}\right)+\frac{2}{3}\beta\mathbf{n}\cdot(\omega_2\times\mathbf{P})+\frac{7}{10}\frac{E_2+m_2}{m_2}\frac{|d|^2}{|p|^2+|d|^2}(\mathbf{P}\cdot\mathbf{n})(\omega_2\cdot\mathbf{n})$$
$$-\frac{\alpha}{2}\left(1+\omega_2\cdot\mathbf{n}\right)n^in^jQ_{ij}+\frac{1}{6}\left[\alpha n^in^jn^k-\beta n^in^jn_s\varepsilon^{lsk}\omega_{2l}-\gamma n^in^j\omega_2^k-\frac{E_2+m_2}{m_2}\frac{|d|^2}{|p|^2+|d|^2}n^in^jn^k(\omega_2\cdot\mathbf{n})\right]R_{ijk}, \quad (13)$$

where α, β, and γ are defined as in Eq. (2), except that s and p are replaced by p and d, respectively. In Eq. (13) we have introduced the spin-3/2 polarization vector $P^\mu = (0,\mathbf{P}) = (0, 2\omega_1)$, the symmetric quadrupole polarization tensor $Q^{00} = Q^{0j} = 0, Q^{ij} = 2\omega_1^i\omega_1^j$, and the octupole polarization tensor $R^{000} = R^{0ij} = 0$, $R^{ijk} = 9\omega_1^i\omega_1^j\omega_1^k$.

3. Conclusions

Equation (13) is the analogous result, for the nonleptonic decays of a spin-3/2 hyperon, to the Lee-Yang formula for the nonleptonic decays of spin-1/2 hyperons [2]. The more complicated result for spin-3/2 is a consequence of the more complex structure of the spin-3/2 particle. Now, we consider two cases of interest. First, if we assume that the spin-3/2 particle is not initially polarized, Eq. (13) reduces to

$$R = 1+\alpha\widehat{\omega}_2\cdot\widehat{n}, \quad (14)$$

showing that the spin-1/2 particle has longitudinal polarization α, in exact parallelism to the nonleptonic decays of spin-1/2 hyperons. Second, if the final baryon is not polarized, Eq. (13) now gives

$$R = 1+\frac{7}{10}\alpha\mathbf{P}\cdot\mathbf{n}-\frac{1}{2}\alpha n^in^jQ_{ij}+\frac{\alpha}{6}n^in^jn^kR_{ijk}$$
$$= 1+\alpha\left[\frac{7}{10}\mathbf{P}\cdot\mathbf{n}-\frac{1}{4}(\mathbf{P}\cdot\mathbf{n})^2+\frac{3}{16}(\mathbf{P}\cdot\mathbf{n})^3\right]. \quad (15)$$

exhibiting a more complicated asymmetry in the emission of the final baryon than in the spin-1/2 case.

Acknowledgments

The authors thanks partial support from Comisión de Operación y Fomento de las Actividades Académicas del IPN.

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