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# Measurement of forces between two parallel conducting cylinders

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We describe the experimental arrangement and method to measure the force between pairs of parallel conducting cylinders at different separations and with fixed potential differences. The results of the measurement for diverse values of the geometrical and electrical parameters are reported. The measured forces are compared with their values evaluated on the basis of Ref. 1, finding a good agreement for cylinders of the same length, and recognizing the edge effects for cylinders of different lengths.

*Keywords:* Conducting cylinders; electrostatic force.

Se describe el arreglo experimental y el método para medir la fuerza entre pares de cilindros conductores paralelos a diferentes separaciones y con diferencias de potencial fijas. Se reportan los resultados de las mediciones para diversos valores de los parámetros geométricos y eléctricos. Las fuerzas medidas se comparan con los valores calculados con base en la Ref. 1, encontrándose un buen acuerdo para cilindros de igual longitud, y reconociendo los efectos de orilla para cilindros de diferentes longitudes.

*Descriptores:* Cilindros conductores; fuerza electrostática.

PACS: 41.10 Dq

## 1. Introduction

The experimental methods of this work may be of interest and useful for students of a course of electricity at the introductory university level. The physical background of the general problem was discussed in a previous paper "Forces between two uniformly charged cylinders versus forces between two conducting cylinders" [1-2], and such students should be made aware of and appreciate qualitatively the difference between both electrical situations. When students take the corresponding junior/senior level course they may also understand the theoretical quantitative treatment of the conducting cylinder problem. Hopefully this work will be useful for teachers and students of both types of courses to understand the measuring and evaluation of such forces.

In this article, we report the measurement of forces between pairs of parallel conducting cylinders placed at different separations and with fixed potential differences. This experimental work complements the work of Ley Koo and Monsivais on the evaluation of such forces [1]. Correspondingly, Sec. 2 contains a description of the experimental arrangement and the method to implement the measurement of the electrostatic force between the cylinders. In Sec. 3, the measured data are presented for different values of the geometrical and electrical parameters involved, and they are compared with the results evaluated according to [1]. Sec. 4 consists of a discussion of such a comparison, showing that there is a good agreement between the measured and evaluated forces for cylinders of the same length, and recognizing that edge effects manifest themselves for cylinders of different lengths.

## 2. Experimental arrangement and method of measurement

The experimental arrangement for measuring the electrical forces between parallel conducting cylinders maintained at fixed potential differences, involved electrical, aligning and force measuring instrumentations, as sketched in Fig. 1 and described next. A Pasco ES-9070 Kilovolt power Supply with an output varying from 0 to 6 Kilovolts provided the potential difference between the conducting copper cylinders. A cathetometer, with a length of 50 cm and an accuracy of ±0.001 cm, was used to establish the horizontality and alignment of the cylinders and to measure their separation. An Ohaus 300D electronic balance, with a capacity of 30 g, an accuracy of ± 0.001 g and a reproducibility of ± 0.007 g, was used to measure the electrostatic force between the cylinders.

Some of the details about the geometrical, electrical and mechanical arrangements and measurements are also described following the successive steps of the experiment. Cylinders with different combinations of lengths h = 6.8, 9.9 and 13.6 cm and diameters 2R = 0.025, 1.59 and 3.5 cm were used. One of the cylinders was hung from a horizontal glass rod using two thin # 32 copper wires; the other cylinder is

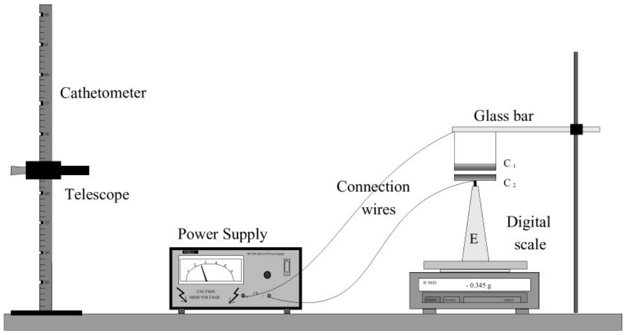

FIGURE 1. Experimental arrangement for the measurement of forces between two parallel conducting cylinders.

supported by a 20 cm high dielectric base resting on the plate of the balance (Fig. 1). The upper and lower cylinders are connected to the positive and negative terminals of the power supply, respectively, using # 32 copper wires. The voltage applied to the cylinders was measured with a high voltage tip coupled to a Fluke 79 digital multimeter; the chosen values were $V$ = 7.5, 10.9, 14.2, 17.5 and 20.5 statvolts. The balance was placed on a concrete base in order to eliminate any vibrations, and it had a servomechanism to keep its plate fixed. The horizontality of the lower cylinder was checked with a level and verified with the axes of the cathetometer's lens. The error for misalignment was estimated to be of the order of ± 0.05 cm. The parallelism of the upper cylinder for each of its successive positions was also verified with the axes of the cathetometer's lens. The cathetometer was used to measure the closest distance $\Delta$ between the upper and lower cylinders, leading to the distance $d = \Delta + R_1 + R_2$ between their respective axes. Before the voltage is applied to the cylinders, the reading of the balance is adjusted to zero, 0.000 g; after the voltage $V$ is applied by turning on the power supply, the attraction between the cylinders produces a negative reading $M$ in grams. The numerical value of the force of attraction in dynes is obtained as the product of this reading times the local value of the acceleration of gravity, $g = 978\ cm/s^2$ for Puebla City. If the cylinders are not properly aligned, the upper cylinder moves and the balance reading oscillates. When the alignment is achieved, the upper cylinder remains at rest and the balance reading is stable after a situation of electro-mechanical equilibrium is reached. This provides a criterion about the quality of the alignment. Each time it was satisfied, the reading of the balance and the separation between the cylinders $\Delta$ indicated by the cathetometer's vernier were written down. The separation between the cylinders was systematically changed through the raising of the upper one by shortening the wires which maintain it hung from the glass rod.

## 3. Measured and evaluated forces

This section contains the measured data about the geometrical and electrical configurations of different pairs of cylinders and the corresponding electrostatic forces between them. The values of these forces are also computed using the formulas of Ref. 1, and a comparison between the measured and computed forces is made.

The electrostatic field between two conducting parallel cylinders of infinite length was evaluated in Ref.1 using bipolar coordinates [2]. Instead of the radii $R_1$ and $R_2$ of the cylinders and the distance $d$ between their respective axes, the geometrical configuration can be described in terms of the polar distance $2a$ and the respective circular coordinates $\eta_1$ and $\eta_2$. The connection between both sets of parameters is given by the equations

$$R_1 = -a\text{csch}\eta_1, \qquad R_2 = -a\text{csch}\eta_2, \tag{1}$$

$$d = a\text{coth}\eta_2 - a\text{coth}\eta_1, \tag{2}$$

where $-\infty < \eta_1 < 0$ and $0 < \eta_2 < \infty$. Elimination of $\eta_1$ and $\eta_2$ in Eqs. (1) and (2) leads to the complementary connection,

$$a = \frac{\sqrt{(d+R_1+R_2)(d+R_1-R_2)(d-R_1-R_2)(d-R_1+R_2)}}{2d} \tag{3}$$

In the limit of vanishing radii, $R_1 \rightarrow 0$ and $R_2 \rightarrow 0$, Eqs. (1) give $\eta_1 \rightarrow -\infty$ and $\eta_2 \rightarrow \infty$, and Eqs. (2) and (3) give $d \rightarrow 2a$.

The advantage of using bipolar coordinates in the analysis of the electrostatic problem of the two cylinders maintained at fixed potentials $V_1$ and $V_2$ is that the electrostatic potential is a linear function in the circular coordinate, and correspondingly the electric intensity field and the surface charge densities on the cylinders are well defined and simple functions in the bipolar coordinates. This, in turn, allows the easy evaluation of the force between the cylinders. Specifically, the force of cylinder 1 on each portion of length $h$ of cylinder 2 takes the form [1]

$$\vec{F}_{1\rightarrow 2} = -\frac{\hat{k}h}{4a}\left(\frac{V_2 - V_1}{\eta_2 - \eta_1}\right)^2 \tag{4}$$

The application of Eq. (4) to evaluate the force between two parallel conducting cylinders requires the values of the length $h$, the potential difference $V_2 - V_1 = V$ applied to the cylinders, and the values of the polar distance $2a$ and the circular coordinates $\eta_1$ and $\eta_2$ obtained through Eqs. (3) and (1) from the values of $d, R_1$ and $R_2$.

The work of Ref.1 assumes infinitely long cylinders, and it is reasonable to question the applicability of its results to the analysis of the measurement of forces between finite length cylinders. The comparison of the measured and computed values of the forces provides a basis to answer such a questioning.

In practice, the measurements were carried out for pairs of cylinders of dimensions $(h_1, R_1)$ and $(h_2, R_2)$ and applied voltages $V$ for the sets of values described in Sec. 2, and different separations between the cylinders $d = \Delta + R_1 + R_2$.

Table I presents the numerical values of the geometrical parameters –lengths and radii of pairs of cylinders– and the potential difference between them. The distances between their axes, the readings of the digital balance and the corresponding forces, when the electromechanical equilibrium is reached, are the measured data. The experimental values of the reduced force $F/V^2$ and their theoretical counterparts from Eq. (4) can be directly compared in the last two columns.

The same data are presented in graphical form in Figs. 2 - 4 for cylinders of equal lengths and radii, and of equal lengths and different radii. The overall closeness of the circles with their error bars and the continuous curves illustrates that the measured reduced forces and their values evaluated from Eq. (4) are in good agreement for cylinders with the same lengths.

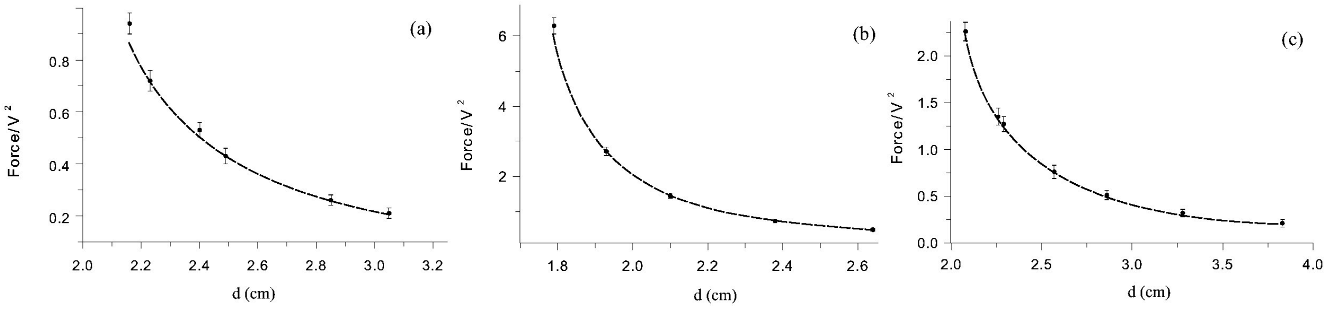

FIGURE 2. Measured (circles with error bars) and evaluated (continuous curves) reduced forces $F/V^2$ versus distance for pairs of cylinders with the same radii ($R_1 = R_2 = 0.795cm$) and the same lengths a) $h_1 = h_2 = 6.8$ cm, b) $h_1 = h_2 = 9.9cm$ and c)$h_1 = h_2 = 13.6cm$.

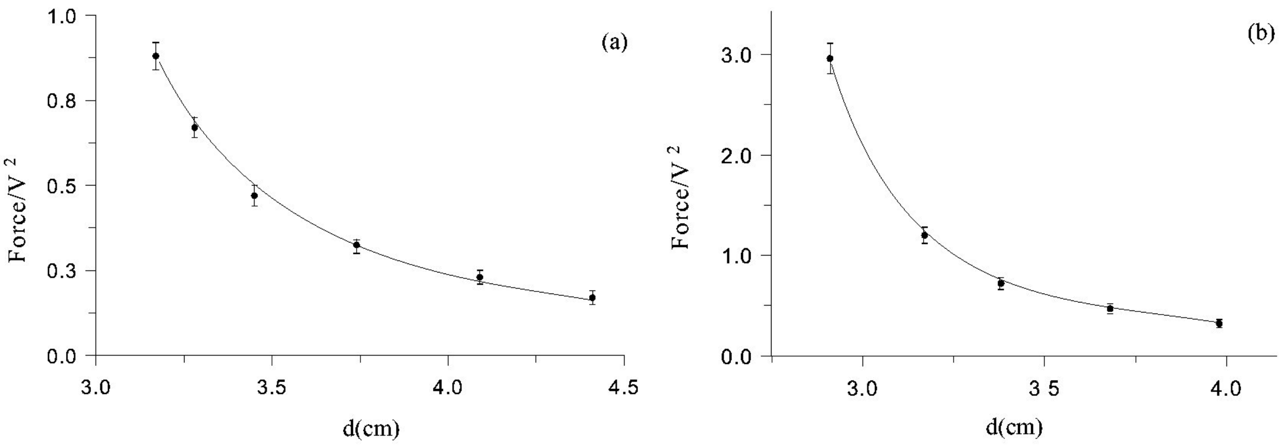

FIGURE 3. Measured (circles with error bars) and evaluated (continuous curves) reduced forces $F/V^2$ versus distances for pairs of cylinders with different radii ($R_1 = 1.75$ and $R_2 = 0.795cm$) and the same lengths a) $h_1 = h_2 = 6.8$ cm, and b) $h_1 = h_2 = 9.9cm$

TABLE I. Cylinder lengths $h_1$ and $h_2$, radii $R_1$ and $R_2$, and their potential differences $V$ (statvolts). Measured separations $d$ (cm), digital balance readings $M$ (g) and corresponding forces $F$ (dynes). Experimental and theoretical, Eq. (4), values of reduced force $F/V^2$.

| $h_1 = h_2$ | $R_1$ | $R_2$ | $V$ | $d$ | $M$ | $F$ | $F/V^2$ | $F_t/V^2$ |
|---|---|---|---|---|---|---|---|---|
| 6.8 | 0.795 | 0.795 | 20.5 | 2.16 | 406 | 397 | 0.94 ±0.04 | 0.867 |
| | | | | 2.23 | 308 | 301 | 0.72±0.04 | 0.717 |
| | | | | 2.40 | 226 | 221 | 0.53 ±0.03 | 0.503 |
| | | | | 2.49 | 185 | 181 | 0.43 ±0.03 | 0.426 |
| | | | | 2.85 | 110 | 108 | 0.26 ± 0.02 | 0.254 |
| | | | | 3.05 | 90 | 88 | 0.21 ± 0.02 | 0.203 |
| 9.9 | 0.795 | 0.795 | 17.5 | 1.79 | 1970 | 1927 | 6.30 ± 0.2 | 6.016 |
| | | | | 1.93 | 849 | 830 | 2.70 ± 0.1 | 2.713 |
| | | | | 2.10 | 453 | 443 | 1.45 ± 0.07 | 1.466 |
| | | | | 2.38 | 230 | 225 | 0.73 ± 0.04 | 0.756 |
| | | | | 2.64 | 153 | 150 | 0.48 ± 0.04 | 0.494 |
| 13.6 | 0.795 | 0.795 | 14.2 | 2.08 | 466 | 456 | 2.30 ± 0.1 | 2.159 |
| | | | | 2.26 | 278 | 272 | 1.35 ± 0.09 | 1.329 |
| | | | | 2.29 | 262 | 256 | 1.27 ± 0.08 | 1.262 |
| | | | | 2.57 | 156 | 153 | 0.76 ± 0.07 | 0.747 |
| | | | | 2.86 | 105 | 103 | 0.51 ± 0.05 | 0.505 |
| | | | | 3.28 | 67 | 65 | 0.32 ± 0.04 | 0.324 |
| | | | | 3.83 | 43 | 42 | 0.21 ± 0.04 | 0.209 |
| 6.8 | 1.75 | 0.795 | 20.5 | 3.17 | 377 | 369 | 0.88 ± 0.04 | 0.883 |
| | | | | 3.28 | 290 | 284 | 0.67 ± 0.03 | 0.691 |
| | | | | 3.45 | 202 | 197 | 0.47 ± 0.03 | 0.501 |
| | | | | 3.74 | 140 | 137 | 0.32 ± 0.02 | 0.328 |
| | | | | 4.09 | 97 | 95 | 0.23 ± 0.02 | 0.220 |
| | | | | 4.41 | 73 | 71 | 0.17 ± 0.02 | 0.165 |
| 9.9 | 1.75 | 0.795 | 14.2 | 2.91 | 612 | 598 | 2.90 ± 0.1 | 2.861 |
| | | | | 3.17 | 249 | 243 | 1.20 ± 0.08 | 1.264 |
| | | | | 3.38 | 149 | 146 | 0.72 ± 0.06 | 0.814 |
| | | | | 3.68 | 96 | 94 | 0.47 ± 0.05 | 0.513 |
| | | | | 3.98 | 67 | 65 | 0.32 ± 0.04 | 0.359 |
| 6.8 | 0.0125 | 0.795 | 17.5 | 1.17 | 89 | 87 | 0.28 ± 0.03 | 0.291 |
| | | | | 1.24 | 68 | 66 | 0.21 ± 0.03 | 0.229 |
| | | | | 1.39 | 50 | 49 | 0.16 ± 0.03 | 0.153 |
| | | | | 1.51 | 39 | 38 | 0.12 ± 0.02 | 0.119 |
| | | | | 1.65 | 33 | 32 | 0.10 ± 0.02 | 0.094 |
| | | | | 1.81 | 24 | 23 | 0.07 ± 0.02 | 0.074 |

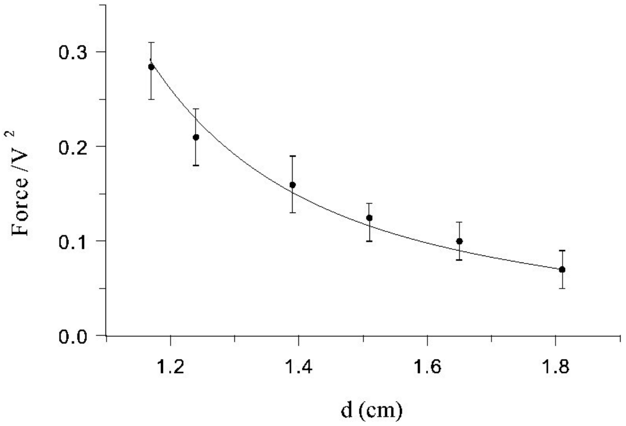

FIGURE 4. Measured (circles with error bars) and evaluated (continuous curves) reduced forces $F/V^2$ versus distance for a pair of cylinders with different radii ($R_1 = 1.75$ and $R_2 = 0.0125 cm$) and the same lengths a) $h_1 = h_2 = 6.8$ cm.

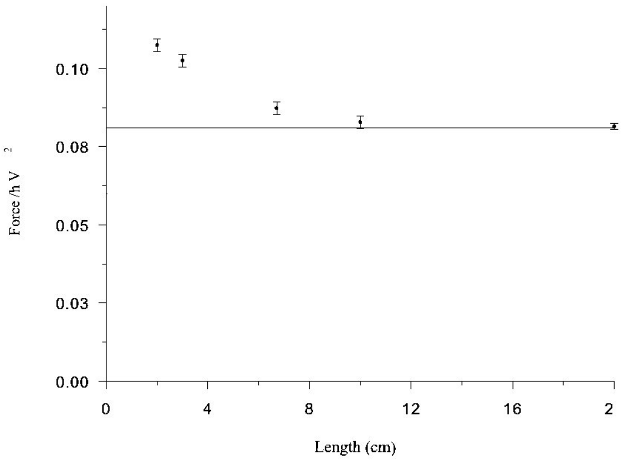

FIGURE 5. Measured (circles with error bars) reduced force per unit length $F/V^2h$ versus length $h$ for the force exerted by a cylinders of radius 0.795cm and length 20 cm on cylinder of radius 0.795 cm and lengths $h$ for a separation $d = 2.35 cm$. The horizontal line gives the value of the force for $h = 20$cm.

## 4. Discussion

This work is concerned with the measurement of the forces between two parallel conducting cylinders when an electric potential difference is applied between them. Sec. 2 described the experimental arrangement to achieve the desired electromechanical equilibrium and to measure the geometrical, electrical and dynamical quantities of interest. In section 3 the corresponding data were presented through Table I and Figs. 2-4, including their comparison with the forces evaluated on the basis of Ref. 1. The comparison shows a good agreement between the experimental and theoretical values for cylinders of equal lenghts.

It is important to recognize that the work of Ref. [1] is valid for cylinders of infinite length. However, the above comparison shows that it is also valid for the description of the transverse electrostatic field of parallel cylinders of equal lengths.

Complementary measurements of the forces between cylinders of different lengths show departures from Eq. (4) due to edge effects. In fact, Fig. 5 shows the reduced force per unit length $F/V^2h$ exerted by a cylinder of length 20 cm on shorter cylinders for a fixed separation distance. The force per unit length increases as the cylinders become shorter, compared with the value for equal length cylinders.

1. E. Ley Koo and G. Monsivais, *Rev. Mex. Fis.* **41** (1995) 610.
2. E. Ley Koo and G. Monsivais, *Rev. Mex. Fis.* **45** (1999) 108.