Students' difficulties with tension in massless strings. Part I.

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Many students enrolled in the introductory mechanics courses have learning difficulties related to the concept of force in the context of tension in massless strings. One of the potential causes could be a lack of functional understanding through a traditional instruction. In this article, we show a collection of this kind of students' difficulties at the New Mexico State University, at the Arizona State University, and at the Independent university of Ciudad Juarez in Mexico. These difficulties were collected during an investigation conducted not only in lab sessions but also in lecture sessions. The first part of the investigation is developed in the contexts of proximity and the length of strings.

Keywords: Tension; forces in strings; learning difficulties; force as a tension.

Muchos estudiantes de los cursos introductorios de mecánica presentan serias dificultades para comprender el concepto de fuerza como vector en el contexto de la tensión en cuerdas de masa despreciable. Una de las posibles causas es la falta de entendimiento funcional desarrollado durante las clases fundamentadas en una enseñanza tradicional. En este articulo, presentamos una serie de este tipo de problemas de aprendizaje que tienen los alumnos de los cursos de física clásica y estática en la Universidad Estatal de Nuevo México, en la Universidad Estatal de Arizona y en la Universidad Autónoma de Ciudad Juárez. Estas dificultades de aprendizaje se obtienen durante una investigación conducida tanto en laboratorios como en el salón de clases. En la primera parte de la investigación se tratan los contextos de proximidad y longitud de las cuerdas.

Descriptores: Tensión; fuerzas en cuerdas; dificultades de aprendizaje; fuerza como una tensión.

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1. Introduction

For almost all students enrolled in an introductory physics course, the initial sequence of topics is kinematics, followed by dynamics. This first exposure to physics has Newton's second law –a vector equation– as its central theme. For this reason, students' perception of what physics *is*, and what it means to *do physics*, are strongly influenced by this topic.

In the ideal case, students will learn from this topic that fundamental principles of physics are powerful general ideas that have broad applicability. Too often, however, students fail to see the connections between the ideas that are presented. Rather than view physics as a subject grounded in a few far-reaching fundamental ideas, they instead gain an impression that the subject is a collection of context-specific [1] equations that must be memorized.

Most instructors of introductory physics courses recognize that thinking about physical quantities as vectors is difficult for students Flores, Kanim and Kautz [2]. Even when instructors consistently model Newton's second law problem solutions by starting with free-body diagrams, many students avoid these diagrammatic tools. There is a tendency, even among fairly capable students, to jump to force components immediately, and to resort to memorizing what these components are in specific cases rather than deriving them from the geometry of the situation. Therefore, students have understanding difficulties with problems that require several steps along the solution process. These problems are called "multiple-step" problems.

In the process of an investigation conducted by Flores [3] into student use of vectors, he observed several difficulties with vectors. This observation motivated an investigation into student understanding of tension. In this article, we describe our observations into students' difficulties with tension.

As shown in the results from the *gymnast question* asked by Flores, Kanim and Kautz [2], it is often the case that students do not acquire a sufficient understanding of tension as a vector concept. Most students (70%) concluded that the tension in the left rope is one-half of the weight of the gymnast. Most of them gave the reasoning that the tensions in the ropes are equal to each other because the angles of the ropes are the same. Implicit in this response is an assumption that the scalar sum of the two tensions equals the weight. This response neglects the vector nature of tension. In order to make sense of forces, students need knowledge of the behavior of specific forces and the rules or assumptions used in physics to solve problems involving these kinds of forces.

On other questions they noticed that students were unable to identify essential features that determine tension. In this sense, we decided to conduct an investigation into student understanding of Newton's second law in the context of tension forces along a "massless" string.

The questions we hoped to answer with our investigation are: (1) Do students recognize the vector nature of tension force?; (2) Do students recognize that tension along a string is the same regardless of the statics or dynamics situation of the string?; (3) Do students recognize that the tension in a massless string does not depend on the length of the string?

2. Previous research

Common-sense beliefs about mechanics are often serious obstacles to succeed in physics. Halloun and Hestenes [4] developed a multiple-choice mechanics diagnostic test, called the *Force Concept Inventory* (FCI) that measures the prevalence of some of these common-sense beliefs in mechanics. Results reported are from interviews with 22 students and from a pretest administered to 478 students. About 14% of students answered in a manner consistent with a belief that a particle subjected to a constant force moves with a constant speed. Of these students about 40% assumed that the increment in speed is proportional to the force and/or the distance traveled. Almost one half of the students' answers suggest a belief that the time interval needed to travel a specified distance is inversely proportional to the magnitude of the force.

For the purposes of our study, it is important to note that even though the *Force Concept Inventory* has become the most common measure of conceptual understanding of mechanics, very little understanding of vectors is required to successfully answer the questions on it. While there is an extensive body of research into student understanding of forces and of acceleration, most of these investigations do not explicitly look at student understanding of vectors.

To identify common student conceptual errors to recognize the existence of passive forces such as the tension in a string, Sjoberg and Lie [5] of the University of Oslo administered a written questionnaire to over 1000 secondary school students, future teachers, university students and physics graduate students.

Figure 1 shows two pendulums, one stationary and one swinging through its equilibrium position. Sjoberg and Lie asked students to indicate the forces acting on both pendulums. Results indicated that about 50% of the secondary-school students with one year of physics omitted the tension in the string. About 40% of the future teachers and about 10% of the graduate students omitted this force as well. A great number of students included a force in the direction of the motion of the swinging pendulum.

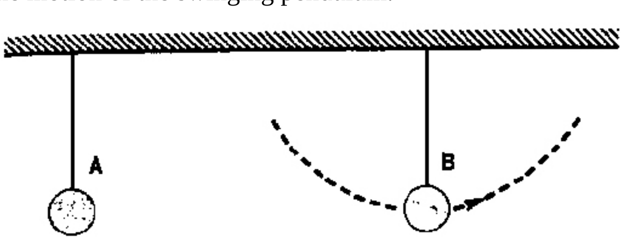

FIGURE 1. Experiment set used by Sjoberg and Lie to probe student difficulties with forces.

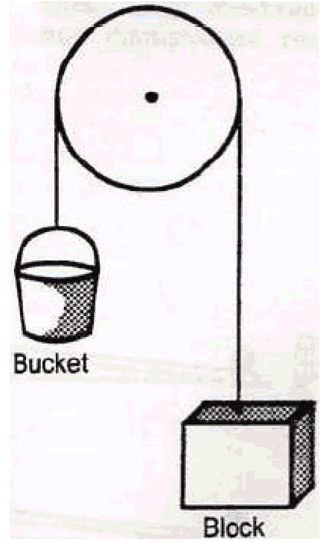

FIGURE 2. Experiment set used by Gunstone and White. The bucket and the block are suspended from a bicycle wheel.

As part of an investigation into students understanding of gravity, Gunstone and White [6] asked 463 students to compare the weight of a bucket with the weight of a block when they are hanging from a string stretched around a pulley as shown in Fig. 2. About one-half of the students concluded correctly that the weights are equal. About one-fourth stated that the block is heavier. The most common reason for this response was that "the block is nearer to the floor." There was a version of other reasons given. For example, "In the string used to link both the bucket and the block together over the pulley, tension exists in both its ends. At the end towards the bucket, the tension is less than at the end towards the block. This then causes the block to pull itself down thereby raising the bucket."

Arons [7] made the observation that "massless strings are a source of significant conceptual trouble for many students." He also states that "students have no intelligible operational definition of *massless;* they fail to see why the forces of tension should have equal magnitude at either end; they proceed to memorize problem-solving procedures without understanding what they are doing."

This observation led to an investigation by McDermott, Shaffer and Somers [8] into some specific student difficulties with tension in the context of the Atwood's machine. Figure 3 shows a physical situation used in this investigation, and Fig. 4 shows the correct free-body diagram for the two hanging masses. The string and the pulley are massless. In interviews, most students predicted that the heaver mass would fall and the lighter mass would rise. Although all recognized that the tension in the string acting on block A is greater than the mass of this block, on the free-body diagrams many showed different magnitudes for the tension exerted by the string on the two blocks.

As a part of the same investigation, a written question based on Fig. 5 was administered to students in three calculus-based courses. Students were asked to compare the magnitude of the tension at the middle of the strings in cases (a) and (b). Only about half of the students predicted that the two strings would have the same tension. Many students responded that the tension in the string attached to the two blocks would be twice that in the other string. Two common difficulties found were: 1) The belief that tension is the sum of the forces exerted at the two ends; and 2) The belief that an inanimate object, such as a wall, does not exert a force on a string.

McDermott, Shaffer and Somers concluded that student performance on simple qualitative questions that were asked after lecture instruction suggested that traditional instruction on the Atwood's machine did not improve understanding of dynamics. Practice in only one context such as the Atwood's machine is not enough to develop a functional understanding of the concept of tension.

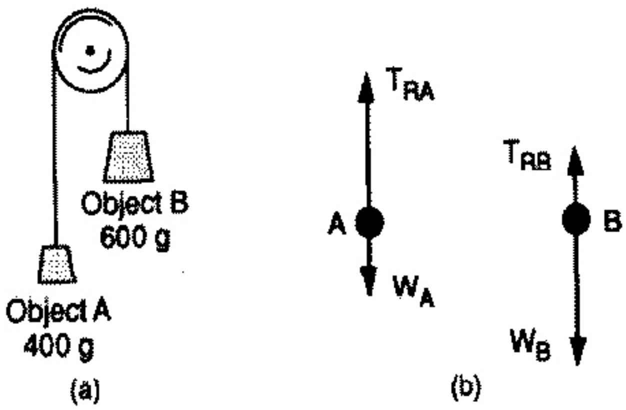

FIGURE 3. Physical system used by McDermott, Shaffer and Somers. a) Original Atwood's machine. b) Typical incorrect free-body diagram drawn by students to represent the forces exerted on blocks A and B.

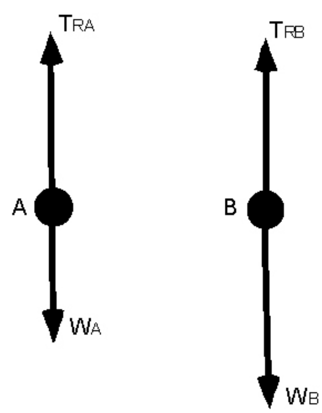

FIGURE 4. Correct free-body diagram for the hanging masses.

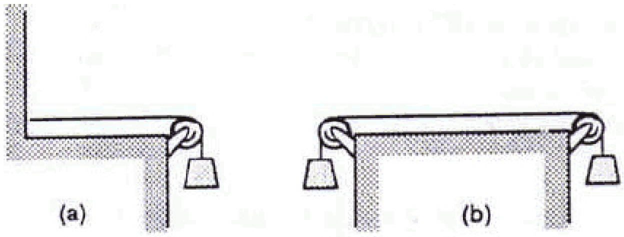

FIGURE 5. Physical situation used by McDermott, Shaffer and Somers. Students were asked to compare the tension in the two strings in cases a and b.

3. Research techniques

There are two primary data sources that we use to assess student understanding and to learn about students' ideas about physics topics and about the prevalence of these ideas in a given student population. These are individual student responses to questions in one-on-one interviews and student responses to written questions. We describe each of these in turn.

3.1. Written questions

Our primary source of data for our investigation was student responses to written questions. These questions were asked on homework (both laboratory and lecture), as laboratory pretests, and on classroom quizzes and examinations. Since we are primarily interested in students' conceptual understanding of physics, the questions we ask are primarily qualitative rather than quantitative. Student responses to these questions are typically analyzed and categorized on the basis of response and of the reasoning given for that response.

In our analysis of these written questions, we are looking for *patterns* of student responses, either correct or incorrect. These patterns may be patterns of incorrect ideas, a common tendency to focus on irrelevant features, patterns of reasoning, or patterns of procedure. Some features of common student responses that seem to lead to correct responses may then form the basis for curriculum exercises that reinforce productive lines of reasoning. Conversely, other patterns of responses may indicate that there is a need for curriculum that elicits a common misconception or error of procedure and reasoning and then addresses this difficulty.

Physics education researchers have found that certain formats of written tasks are useful at eliciting students' ideas and reasoning. For example, a *ranking task*, presents students with a number of physical situations, and they are asked to rank the magnitude of a physical quantity in the given situation. A *comparison task* is similar except that students are asked to compare only two situations, possibly before and after some physical change. Another task that is useful at eliciting student reasoning is the *conflicting contentions task*, in which students are presented with statements about a physical situation and asked whether they agree with any of them. In general, students are asked to explain the reasoning underlying their responses.

3.2. Interviews

Interviews were conducted at New Mexico State University and by colleagues at Arizona State University. These interviews were audio or videotaped, and the tapes and student written responses were later analyzed. At NMSU, we interviewed students from the introductory calculus-based mechanics courses intended for engineering majors. All of these students were volunteers. The interviews last about 30 minutes. We designed the interviews to probe students' conceptual reasoning. During the interview students were asked questions about selected topics and were encouraged to explain the reasoning behind their responses.

4. Context for research

While the data presented here were collected primarily at New Mexico State University (NMSU), we have collected additional data from the Arizona State University (ASU), and the Independent University of Juarez in Mexico (UACJ). In this investigation, student responses from UACJ have been translated from Spanish to English.

The courses used as information sources for this investigation were:

- NMSU: Physics 215 (Introductory calculus-based mechanics).
- NMSU: Physics 211 (Introductory algebra-based mechanics).
- NMSU: Physics 215 laboratory.
- NMSU: Physics 211 laboratory.
- Arizona State University: Physics 212 (Calculus-based mechanics).
- Independent University of Juarez: General Physics I (Calculus-based mechanics).

Physics 215 is primarily intended for engineering majors. Instruction in introductory calculus-based physics courses at New Mexico State University consists of three 50-minute lectures. The sequence of topics in lecture follows the sequence in most textbooks. There is no recitation section.

Physics 211, the algebra-based physics course, covers more topics than the calculus-based course, but at a less rigorous mathematical level. The majors of the students enrolled in Physics 211 are approximately: 30% Engineering Technology, 30% Biology, 10% Agriculture, 5% Education, and 20% Other/Undeclared.

There is an associated 1-credit laboratory, Physics 211L and Physics 215L, that is required for some majors. About one-half of the students enrolled in the lecture portion of the course also take the laboratory. The 3-hour laboratory is graded separately from the lecture. All of the laboratory sessions are taught by graduate students. In laboratory, students work in small groups on materials intended to strengthen connections between observed phenomena and mathematical formalism, to promote scientific reasoning skills, and to foster conceptual understanding. Instead of a laboratory report, students are assigned laboratory homework intended to reinforce and extend concepts underlying the laboratory. Students are encouraged to *predict*, *compare* or *rank* variables in physical situations. Most of the laboratory sessions for both the calculus-based and the algebra-based course were based on *Tutorials in Introductory Physics* [9].

5. Traditional and modified instructions

At the University of Juarez, at the Arizona State University and in some courses at NMSU the instruction is characterized as traditional. By traditional instruction, we mean instruction that is similar in emphasis and approach to that found in most introductory classrooms. That is, there is no particular emphasis placed on the topics under investigation in this dissertation, nor is there any modification to the instructional technique used. In addition, the use of numerical and textbook problems on homework and exams. An example of a textbook problem is shown in Fig. 6. In this problem, students are asked to calculate the coefficient of static friction between the ladder and the floor.

Instruction in Physics 121 at Arizona State University all these courses have a 3-hour lecture and a 1-hour recitation section with traditional laboratories. About 90% of the students take laboratory sections. The laboratory is independent of lecture. Assessments in lecture include conceptual problems.

The questions we have asked at the University of Juarez were in a 3-hour calculus-based physics class. A 90-minute

TABLE I. Results for answers of the two-pulley problem.

	NMSU Midterm exam N=90	UACJ Final exam N=105
All tension are equal	42%	34%
D=E>A=B=C	13%	5%
A=B=C>D=E	20%	27%
A=B>C>D>E	8%	0%
A>B>C=D=E	2%	2%

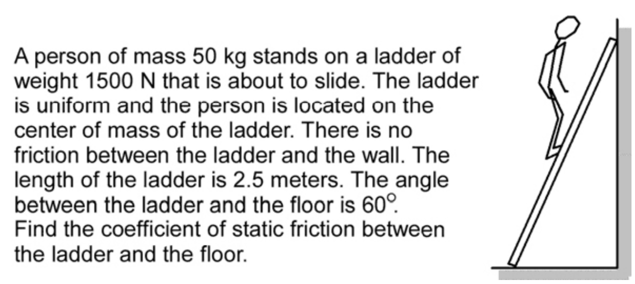

FIGURE 6. Example of problem used in traditional instruction.

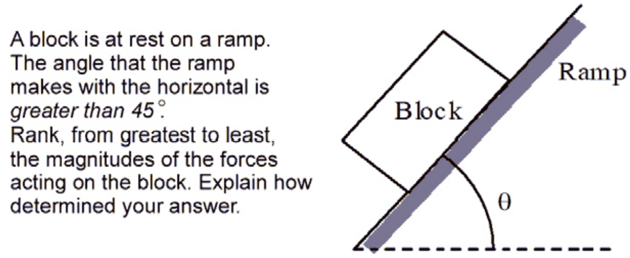

FIGURE 7. Example of question asked in a modified instruction.

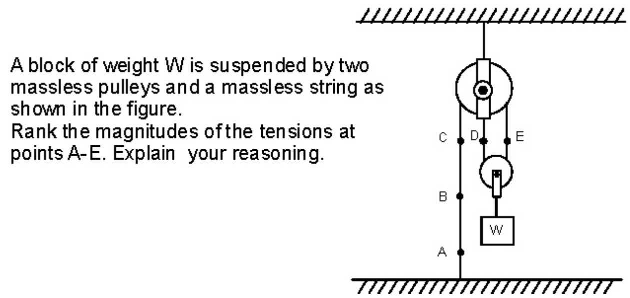

FIGURE 8. Question to probe student understanding of tension on a massless string.

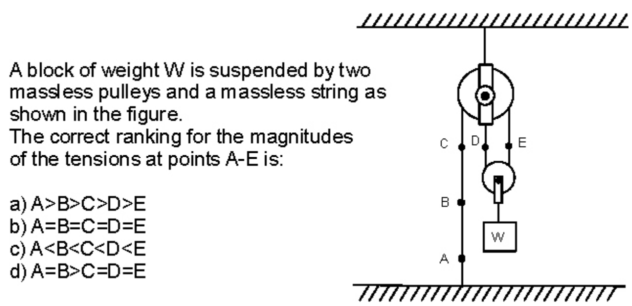

FIGURE 9. Multiple-choice version.

laboratory session per week is required. Laboratory is independent of lecture and mandatory. All laboratory sections are taught by the corresponding instructors of the groups. There is no recitation section. Most of the students are engineering majors. Questions on homeworks and examinations are primarily quantitative.

Most of the coursework at NMSU that we describe as modified was taught by Stephen Kanim. He modified the lecture the lecture section of the course to increase the emphasis on conceptual understanding. Many homework assignments, exams and exercises are composed of conceptual physical problems. The emphasis of the lectures was modified to focus on vector concepts: The course begins with an introduction of vector addition in the context of force, and velocity and acceleration are first introduced in two dimensions to emphasize their vector nature. An example of a problem used in modified instruction at NMSU is shown in Fig. 7. Students are asked to rank, from greatest to least, the magnitudes of the forces acting on the block. An explanation of the reasoning procedure is required. Some of the students draw a free-body diagram of the block and show a vector sum to compare the magnitudes of the forces.

In this article, we describe our identification of student difficulties with vectors and tension, and illustrate how this identification has guided some curriculum development. In addition, this article provides details of our investigation of student difficulties with the vector nature of tension. We have separated these difficulties into categories:

1) Belief that the magnitude of the tension is the same along a massless string;
2) Belief that the magnitude of the tension depends on the length of the string;
3) Belief that the magnitude of the tension is the same in both sides of the pulley;
4) Ideas about compensation; and
5) Belief that the magnitude of the tension depends on the angle of the string (the last three difficulties are shown in Part II of the article).

6. Students' learning difficulties with tension in massless strings

We have asked a number of questions on homework, pretests, and examinations in order to investigate student understanding of tension. Based on these questions we have classified students' difficulties with tension into five categories:

1. Association of tension with the proximity to an object.
2. Association of tension with string length.

6.1. Students' association of tension to proximity

6.1.1.

The question shown in Fig. 8 was given on examinations to 190 students at New Mexico State (in 3 different sections) and to 105 students at the University of Juarez in Mexico. They were asked to rank the magnitudes of the tension at five points in different sections of a massless string that is holding a block. Students were required to explain their reasoning in all but 1 section at NMSU. A correct answer is that since the string is massless, all of the points have the same tension. Figure 9 shows the multiple-choice version of the question.

As shown in Table I, about 40% of students from New Mexico State and one third from Juarez answered correctly. Almost one-half of the students from both universities who answered correctly explained that the tension is the same at the five points because they are on the same string. Thirty five of the 90 students from New Mexico State reasoned incorrectly on the basis of the location of the points along the string. (On the multiple-choice version given to 100 students at NMSU, 37% chose the correct answer.)

Eleven of the thirty five students who answered incorrectly gave reasoning based on the idea that tension depends on the proximity of the points to the pulleys or to the hanging weight. For example one student answered that "*Greater D,E,C,B,A least. Because the point closer to the weight will experience more tension*."

Another student stated that "D=E>C>B>A. D and E are supporting the weight and also have tension. C is next closest to the weight and also positioned at the top of the rope, where it is also being acted upon by tension B, then A is acted upon by tension and finally A." Other students based their reasoning on proximity to the pulley. For example, "Tension at point A is the greatest because it is farthest from the pulleys. Then followed by B, C and E. D has the smallest tension because it is closest to the pulleys." A similar justification was "A>B>C=D=E. A would be greater because it is farthest from the bigger pulley, then B is second. C, D and E are equal because they are at the same distance."

Other students who gave responses based on the proximity to the pulley reasoned that tension was smallest closer to the pulley.

6.1.2.

As part of a laboratory pretest at New Mexico State, 122 students from two sections of algebra-based physics courses and 112 from two sections of calculus-based physics courses were asked to compare the magnitudes of the tension at two points of a string that is holding a piece of metal of mass M shown in Fig. 10.

According to Table II, about one-third (37 of 122) of the students from the algebra-based courses and about one-quarter (28 of 112) of the students from the calculus-based courses gave incorrect responses.

Seventeen of these students based their reasoning on the proximity to the pulley:

> "*The magnitude of the tension at point 2 is greater than at point 1, because it is closer to the pulley. The closer to the top, the harder it is to pull the mass.*"

> "*The magnitude at point 2 is greater than the magnitude at point 1 because the distance from the pulley is less as compared to point 1.*"

> "*Point 2 has greater tension than point 1 because point 2 is closer to the turning point.*"

> "*The magnitude at point 2 is less than the magnitude at point 1 because the magnitude of the distance from the pulley is less at point 2 than at point 1.*"

Nine of these 36 students based their reasoning on the proximity to the weight:

> "*Less than, because the distance between point 1 and the weight is greater.*"

> "*Greater because the length of the string from 2 to the weight is smaller than that from 1 to the weight.*"

Almost all of the students who said that the tension at point 2 is less than tension at point 1 mentioned the force that is pulling the string by the hand: "*Less than, because most of the tension would be close to the hand, and point 1 because the string is being pulled*." Others based their explanations on the idea of the hand as the origin of the force that diminishes farther from it. They seemed to believe that the closer to the hand, the bigger the tension. Below we show these explanations.

> "*Less than, the origin of the force is the hand and point 2 is farther away than point 1*"

> "*Less than because the force exerted by the hand is lost farther up the string*"

6.1.3. Commentary on students' association of tension to proximity

Similar difficulties and reasoning problems with the proximity argument were observed in students' responses to other questions. Some students' responses suggest a belief that tension decreases with increased distance from an active agent such a person holding a rope. In this view, the hand acts as a "force source"; at points farther away from the hand, the force diminishes. Other students focused on then location of the point with respect to the pulley. Some of them seemed to believe that tension along the string is the same only at those points on the same side of a pulley. Other students' answers suggest that tension diminishes with distance from the pulley. Finally, some students reasoned based on distance from the weight to the point in question.

TABLE II. Results for the comparison of tension at points 1 and 2.

	Algebra-based N=122	Calculus-based N=112
$T_2=T_{1(Correct)}$	54%	65%
$T_2>T_1$	27%	18%
$T_2<T_1$	19%	17%

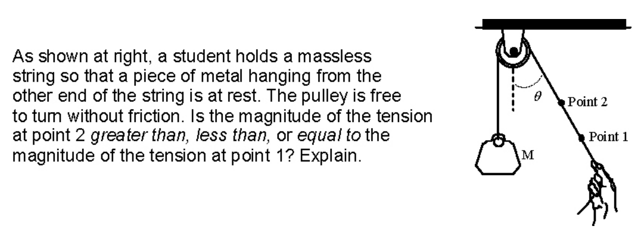

FIGURE 10. Question about the magnitude of tension at two points of a string.

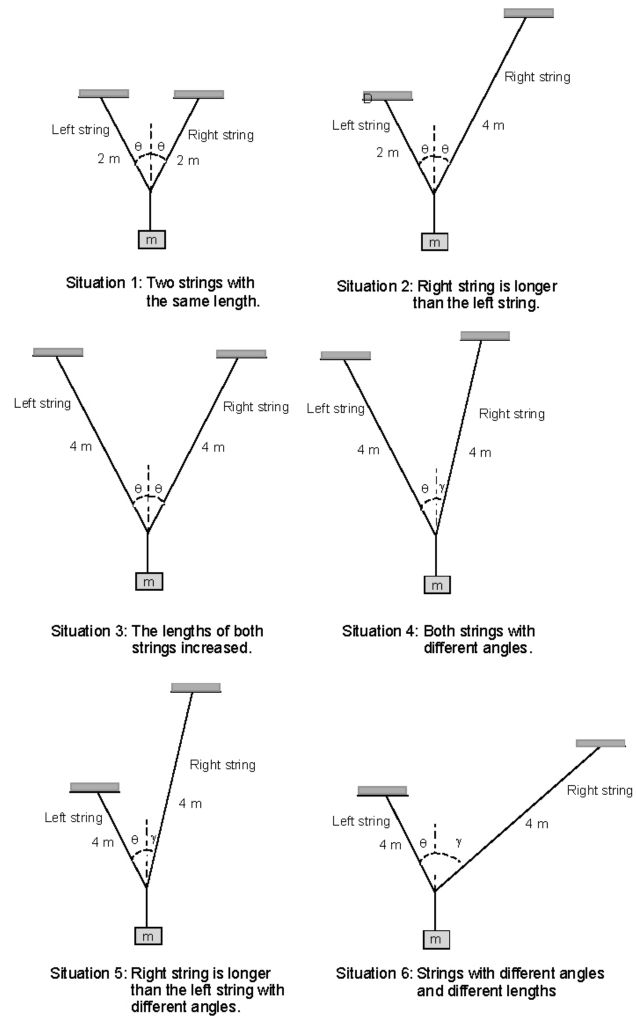

FIGURE 11. Six physical situations for the interviews conducted at NMSU.

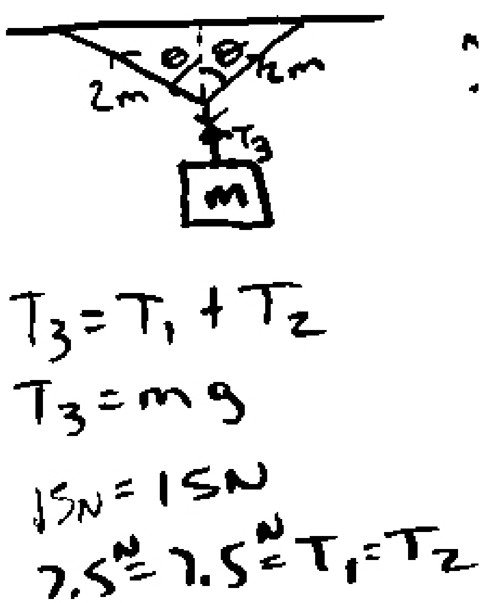

FIGURE 12. Student response during interview with tensions added as scalars.

About one-third of the students in laboratory pretest, and about the same fraction of students in lecture midterm examinations used proximity arguments to answer questions about tension. This fraction does not appear to change significantly as a result of traditional lecture instruction nor as a result of laboratory instruction with no specific focus on conceptual issues related to tension.

6.2. Students’ association of tension with the length of string

Some students gave responses in interviews and to written questions that suggest a belief that the tension in a string or section of a string depends on the length of the string or section.

6.2.1.

At New Mexico State five students were interviewed after the *Forces in Equilibrium* laboratory in which students used a force table to explore forces in static equilibrium. Students were asked to compare the tensions in the two strings shown in the six situations shown in Fig. 11. By analyzing students’ responses, we hoped to isolate the variables that students thought were important in determining tension.

We give examples here that reflect common students’ ideas. Student A was asked to compare the magnitudes of the tensions in the strings in situation 1, and initially gave answers that that treated tension as scalars and used a numerical value for the weight of the hanging mass, as shown in Fig. 12.

During the interview, this student subsequently answered that the magnitudes of the tensions depend on both the angles and lengths of the strings.

> I:“*Is the tension in the left string greater than the tension in the right string?*”
>
> S: “*No, they have the same magnitude*.”
>
> I: “*Does your answer depend on the angles and the lengths of the strings?*”
>
> S: “*It depends on both, the angles and the lengths of the strings. The object is hanging there because both strings have the same force to hold the mass*.”
>
> I: “*If the angles are different?*”
>
> S:“*Let me draw the situation with two different angles.In this case, to keep equilibrium tension must cancel*.”
>
> I: “*In this case, what tension is greater?*”
>
> S: “*I don’t know*.”
>
> I: “*Let go back to situation 1. Do the magnitudes of the tensions depend on the lengths of the strings?*”

S: "*They depend on the angles also. The angles are important for the directions of the vectors. But, here we are changing the angles and the lengths*."

When this student was asked to compare the magnitudes of the tensions in the strings in situation 2, he answered that the longer string had a greater tension. Later he seems to change his mind:

I: "*Assuming that the strings are massless, what tension is greater, the tension in the left string or the tension in the right string?*"

S: "*The right string is going to be, I mean the shorter holds more weight. If we double the length, we have half of the weight.*"

I: "*Then you say that the left tension has double the tension of the \right string?*"

S: "*Yes, the left tension is twice the right tension*."

I: "*So, the ratio between the lengths is the same ratio between the tension. Am I right?*"

S: "*Yes it is.*"

I: "*Why do you conclude that?*"

S: "*Because I remember when I was a child, my brother and I were carrying a box with ice. He was taller then me, and he lifted the box higher than I could, then I felt the box was heaver for me. I am not sure if I am right.*"

Several students who were interviewed gave answers consistent with a belief that string tension depends on length. As shown in the next section, this belief was also expressed by students in response to written questions.

6.2.2.

We designed the question shown in Fig. 13 to measure the prevalence of students' tendency to associate tension with length. In one section of a calculus-based mechanics course this question was asked on a midterm examination; in another section, the same question was asked on a final examination.

About three quarters of the students in both sections answered correctly. An example of a correct response given by a student is: "*Equal to because the angles for the two cases are the same as well as the mass. The length will not affect the tension*." Eight students concluded that the tension in the right string in case A is greater because the string is shorter. These students appear to treat tension (somewhat analogously to pressure), as a quantity that diminishes when it is distributed:

"*The magnitude of the tension in the right string is greater in case A than in case B, because in figure B the right string has more string for the block to distribute its weight*."

"*The tension is spread out over a longer string therefore the magnitude is less in case B*."

"*The tension in the right string in case B would be less than the tension in case A because the weight of the metal is distributed over a longer distance*."

Conversely, nine students answered that the tension in the right string in case A was less because the string was shorter: "*Less than because the length of the string is greater [in case B] than that of case A. They both have the same angle and the same mass*."

6.2.3.

The question shown in Fig. 14 was asked as part of an examination at Arizona State University. Figure 15 shows a correct procedure for comparing tensions based on knowledge that the three forces acting where the strings meet must add to zero. The tension in string A is greater than the tension in string B. The angle the string A makes with the vertical (α)is smaller than the angle that the string B makes with the vertical (β). The lengths of the strings do not affect the tension.

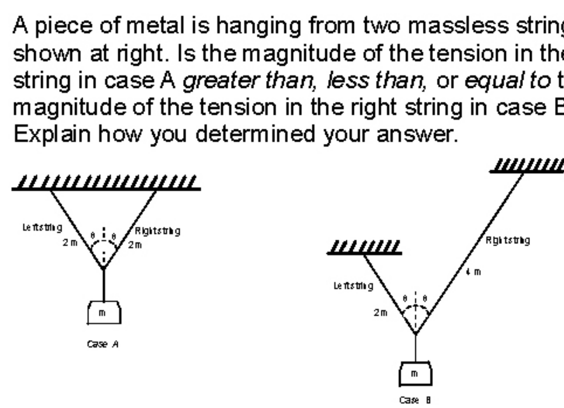

FIGURE 13. Examination question about dependence of tension on length.

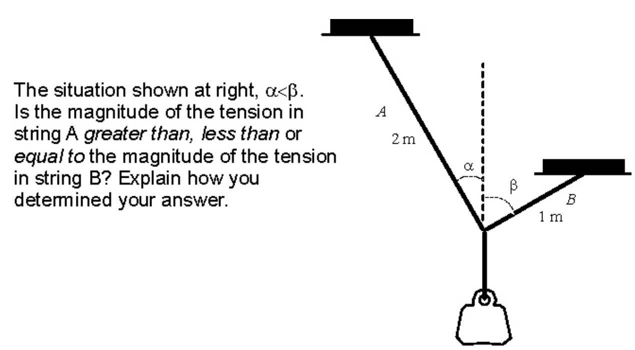

FIGURE 14. Question about tension in strings with different lengths and angles.

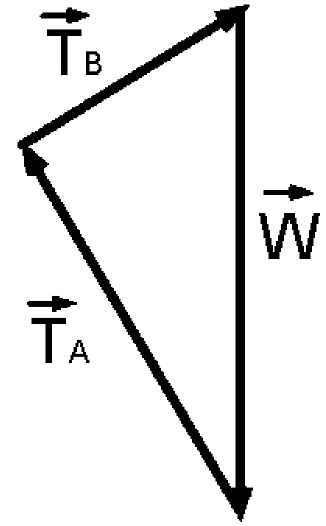

FIGURE 15. A vector sum of the forces acting at the string junction.

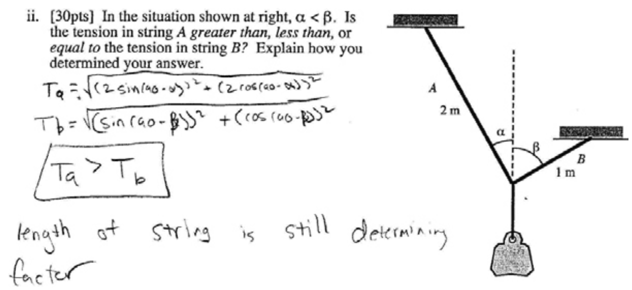

FIGURE 16. Use of Pythagorean Theorem with the lengths of the strings to calculate the magnitudes of the tensions.

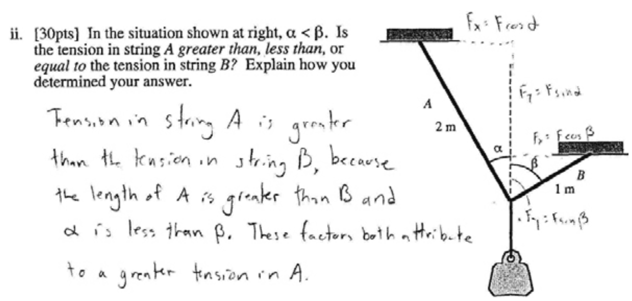

FIGURE 17. Use of the lengths of the strings to compare the magnitudes of the tensions.

Sixty percent of 132 students answered correctly. One-quarter said that the tension in the string A is less than tension in the string B, and only 15% concluded that the magnitudes of both tensions are equal. About 20% (28 students) reasoned based on the lengths of the strings. Eleven of these students gave a correct answer with incorrect reasoning.

Five of these eleven students used the lengths of the strings and the projection of this length in the vertical and horizontal directions to compare the magnitudes of the tensions. They used the Pythagorean Theorem with the lengths of the strings to calculate the tension in both strings. Figure 16 shows one example of this explanation.

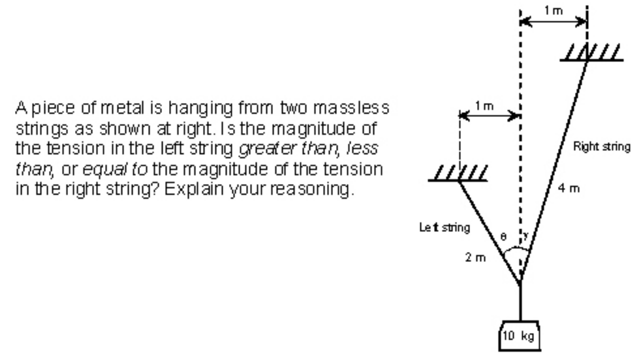

FIGURE 18. Question with strings at different angles.

Some students seemed to believe that the lengths of massless strings affect the magnitudes of the tensions. Some of them related the lengths with the magnitudes of the tensions. They seemed to confuse the diagram of the physical situation with the free-body diagram for the point where the strings meet. Figure 17 shows an example of these responses.

A variation of the question, shown in Fig. 18, was asked to 102 students in two sections of a calculus-based mechanics course as part of a midterm examination. Based on reasoning similar to that for the previous question, the.tension in the left string will be less than the tension in the right string.

About one-half of these students answered correctly and about 15% (15 students) answered that both magnitudes were equal. About 30% of the students gave reasoning based on the lengths of the strings: Twelve of these students answered correctly, and only four responded incorrectly that the tension in the left string is greater than in the right string. Two examples of these responses are shown below.

> "*The tension in the left string is less than the tension in the right string because the right string is longer than the left string. There is more force needed to keep the mass at equilibrium*."

> "*The magnitude in the left string is greater than the the magnitude in the right string since the left string is shorter and since the angle is greater also*."

Thirteen of the 34 students who reasoned based on the lengths of the strings answered that the tensions in both strings are equal. All of them used arguments related to both the angles and lengths. For example, one of them stated that "*Since the angle* θ *is less than* γ *the left tension is less than the right tension, but the length of the string gives the strings an equal ratio in magnitude*."

6.2.4. *Commentary on students' association of tension with the length of string*

Some students seem to have a simple rule that longer strings have more tension.

One the other hand, other students seemed to believe that shorter strings have more tension because the force can be more distributed along longer strings. This idea is sometimes expressed by claiming that tension is more “concentrated” in shorter springs.

Other students used the lengths of the strings as part of a calculation of the magnitudes of the tensions. They seem to confuse the diagram of the strings with a free-body diagram. Other students who concluded that the tension is greater in the longer string seemed to use the diagram of the situation to form conclusions about the relative tensions in the strings.

7. Implications for instruction

Many physics education articles research are related to the reasoning problems student have in introductory mechanics courses. All these authors focus on the process to know the physical concepts learning process. This article is not the exception. This article shares the idea that physics education investigators have about the importance to know this learning problem before the instructional modifications to solve it. Most of us are convinced that we need to understand the reasoning problems student face before the design of the corresponding instructional changes to solve it.

McDermott and the Physics Education Group at the University of Washington [9] have spent more than 15 years trying to understand what wrong reasoning ideas are developed during the learning process. It seems that a very important cognitive noise [10] is produced not only by a traditional instruction, but also by specific instructional changes in classrooms and laboratories. It means that despite the use of learning techniques and sophisticated laboratory activities, most of students do not develop a conceptual structure of many physics topics.

Other investigators as Redish, Saul, Steinberg [1] and Kanim [11] have found several investigation that most of physics teachers have no idea about the incorrect reasoning understanding created by a traditional instruction. In addition, results from their investigations show also that many instructional methods are not base on a scientific knowledge of the cognitive problems students have during the possible physical concepts learning process.

We have designed a laboratory instructional change based on the use of a real representation of the concept of tension and other forces acting on an object. The theoretical base of the design implies that students need a better relationship with the knowledge object, in this case, the concept of tension force in masless strings.

Our original attempts at modification included all of this material in a single laboratory. However, as this investigation proceeded and we recognized additional student difficulties, we expanded this material into a two-laboratory sequence. These modified laboratories are currently the fourth and fifth offered in the semester, and are titled *Forces* and *Addition of Forces*.

Based on our interviews and our analysis of questions asked to probe student understanding, we have designed a laboratory sequence that includes:

1) Identifying the forces acting on an object and drawing free-body diagrams.

2) Exercises focusing on documented student difficulties with weight, tension, normal and friction force.

3) Pencil and paper practice with addition of vectors.

4) Qualitative and quantitative exercises that promote understanding of vector addition of forces.

Each laboratory has an associated pretest, given at the beginning of the laboratory. Students are given 10 minutes to complete the pretest and are given a small amount of credit for completion. The pretest is not graded.

The goals of our modifications to the laboratory were to promote through laboratory exercises:

1) facility at translating from mass to weight, and of appropriate use of these quantities;

2) recognition that the normal force acts in a direction perpendicular to a surface, and does not always have the same magnitude as the weight;

3) recognition that the tension is constant along a massless or light string, and does not change when the direction of a string changes around a pulley; and

4) identification of the friction force as dependent on the normal force and on the type of surface.

FIGURE 19. Equipment set used for tension portion of the *Forces* laboratory.

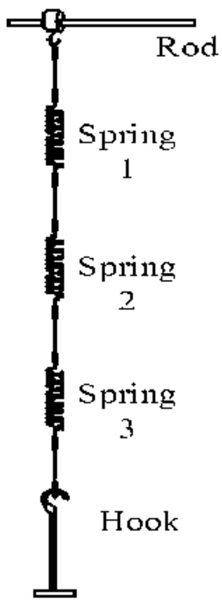

FIGURE 20. Linear spring set.

Here we will describe only those parts of the laboratory related to tension.

In attempting to address student difficulties with tension we used light springs inserted into strings in order to give a qualitative measure of the tension in the string in various

FIGURE 21. Equipment set used in *Addition of Forces* laboratory.

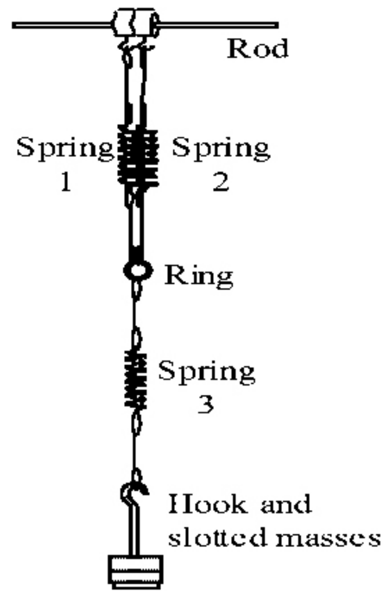

FIGURE 22. Linear spring set used in *Forces* laboratory.

places. Figure 19 shows the set of springs used in *Forces* laboratory. Students are told that the amount the springs stretch provides a measure of the tension in the string. Students are first asked to predict which spring will stretch the most when a mass is suspended, and to explain the basis for their prediction. Students test their prediction by adding slotted masses to the hanging hook as shown in Fig. 20.

The first part of the second laboratory of the two-laboratory sequence, *Addition of Forces*, is intended to give students practice with:

1) addition of vectors,

2) addition of forces included a free-body diagram, and

3) qualitative reasoning about force magnitudes and directions for static cases. The issues we attempted to address are student tendencies to: 1) *Close the loop* when adding vectors; 2) Add vectors as scalars [2]; 3) Inappropriately reason about tension based on angles; and

4) associate tension with string length.

The second portion of the laboratory is intended to give students practice with qualitative and quantitative reasoning about vector addition in static situations. In attempting to address student difficulties with tension, we used springs inserted into strings to give qualitative measure of the tension in the string at various places as in the *Forces* laboratory. Here however, three strings with springs are arranged in a “Y” as shown in Fig. 21. Students are first asked to draw a free-body diagram of a ring connected to three strings arranged in a vertical plane as shown in Fig. 22. Students are then asked to find the net force acting on the ring and to predict the relative lengths of the springs, and to then test their predictions.

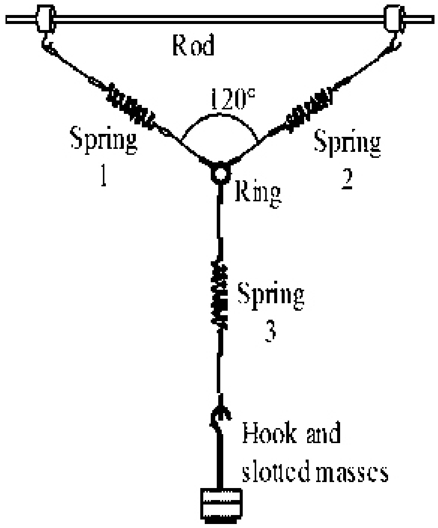

FIGURE 23. "Y" spring set. *Addition of Forces* laboratory.

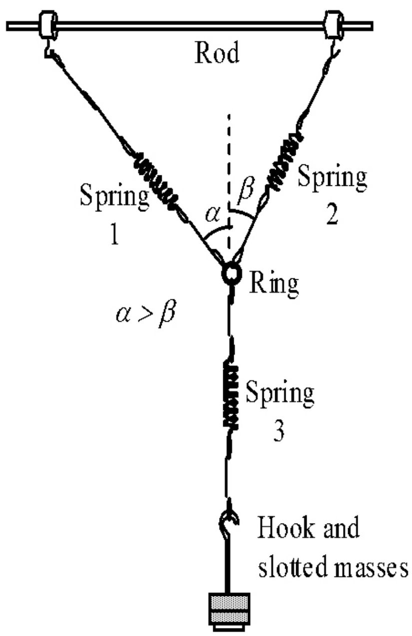

FIGURE 24. "Y" spring set with different angles.

Next, the angle between the upper strings is increased as shown in Fig. 23, and students repeat the procedure. Here they are asked to compare the tension in the strings to each other as well as to the tension in the previous case. Some students might conclude that the tensions in the upper strings do not change because the tensions are equal to one half of the hanging weight. Students are also asked to justify the answer. Some of them might draw a vector sum to compare the magnitudes of the tensions with respect to the angles of the strings.

Finally, this exercise is repeated for a situation where the upper strings make different angles with the vertical, as shown in Fig. 24.

In order to give students practice with the quantitative addition of forces, students then use a force table to observe the forces acting on a ring as shown in Fig. 24. They are first asked to draw a free-body diagram for the ring and to add graphically the forces from the free-body diagram. Students are then asked to predict the magnitude of the resultant vector. They then add force vectors representing the measured forces to find the magnitude and direction of the resultant force they found (which should be zero). Finally, students are asked to find an unknown hanging mass by using a scaled vector sum. As with the *Forces* laboratory, homework is assigned to give students practice at applying the ideas developed in the laboratory.

8. Conclusions

Some of the difficulties of this investigation were found already in another investigation dedicated to students' problems with forces [2]. For example, when students were asked to compare the magnitude of the tension in the left string with a half of the weight of a hanging gymnast, some students answered that the magnitude the tension is equal to one half of the weight no matter the angles that the strings make with the vertical line. It seems that they did not develop a meaningful understanding through a instruction with a conceptual emphasis in forces.

Results from this article show that students' learning difficulties with tension can be found not only in a traditional instruction course but also during a modified instruction. It seems that a conceptual emphasis in physics topics is not enough to establish a formal understanding of tension force in students.

Results also indicate that learning difficulties with tension related to the proximity to an object do not depend on the mathematical basis of the physic source (see Table II). However, Table II indicates that more students through a conceptual basis instruction gave correct answer that students enrolled in a traditional instruction.

Most of the incorrect responses through written questions and interviews seem to indicate that changes in the instruction could be worse in the students' reasoning evolution. It means that if the design of the new instruction does not contain the didactic necessary elements, it is possible to invoke *cognitive noise* [10] (cognitive dissonance) in the understanding process of students. It can be observed when students give a correct answer through an incorrect reasoning, or a correct reasoning with an incorrect answer. Figure 15 shows an example of this reasoning problem.

We have found that after instruction in forces, addition of forces, and tension in particular, many students do not recognize the essential features that determine tension in many physical situations. For example, many students lack the prerequisite knowledge for understanding the force table laboratory. This knowledge is usually implicitly assumed in the design of the laboratory materials.

In interviews and on responses to written questions, we have observed that most students do not treat tension forces as vectors when attempting to answer qualitative questions. They rarely draw free-body diagrams, and even less often add forces to reason about relative magnitudes. Instead, they rely on learned or generated their own rules about the effects of various physical features (string length, angle with the horizontal, proximity to an active agent or a pulley, etc.) on the tensions. We have categorized these rules and given examples or each category based on a subset of the questions that we have asked.

The instructional justification of the two lab sequence to resolve this learning problem is based on the technique *Elicit*, *confront* and *resolve*. This technique states that a mistake is a fundamental didactic element. Students must establish a direct contact with the knowledge object through a real representation of tension force. They realize their mistake (incorrect prediction), and resolve this mistake by invoking the correct concept if it is possible.

We have shown just a brief description of the instructional change in order to address and asses the misunderstanding, common sense conceptual beliefs and understanding problems students have after both, traditional and modified instruction. In addition, similar research results indicate that "there is increasing evidence that after instruction in a typical course, many students are unable to apply the physics formalism that they have studied to situations that they have not expressly memorized. For a meaningful learning to occur, students need more assistance that they can obtain through listening to lectures, reading the textbook, and solving standard quantitative problems" [17].

Finally, we believe that most of introductory physics courses students have learning difficulties with basic physics concepts as tension force. The conceptual development of mathematical object that represent these physics concept determine the cognitive evolution of the knowledge structure of the students. It is important to cite that several investigators related to the science learning have found that most of these students build 10% of a meaningful understanding at most. That is way we and most of the investigators conclude that it is important to understand this learning problem before trying to resolve it.

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