

**Comparison of energy dissipation in downstream of
stepped and baffled spillways of piano key weirs**
**Comparación de la disipación de energía en aguas abajo
de aliviaderos escalonados y desconcertados de
vertederos de tecla de piano**

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Abstract

Heavy seasonal rains over a dam reservoir can lead to huge floods and consequent failure of the dam structure. One way of controlling the adverse effects of heavy rains is to use weirs with larger dimensions such as piano key weirs (PKWs). In spite of raising the overall costs of the project, these weirs are preferred due to their high culvert capacity. PKWs can cause damage to the dam by creating water jets downstream. An effort was made in the present study to examine the energy dissipation due to installation of steps and baffles with different sizes on piano key weirs. The results showed that small baffles and large steps caused maximum energy losses of 85 and 80% in the studied weirs respectively. It was also found that the weir energy dissipation over the spillway was inversely related to its slope angle (the angle between the weir outlet and the vertical wall of the steps/baffles). The other findings were: (1) increasing the flow rate reduced the effect of the steps on the energy loss;

(2) the Weber number was directly proportional to the energy loss; and
(3) the ratio yc/H (critical depth to the weir head ratio) was directly proportional to the energy loss.

Keywords: Piano key weir, energy dissipation, stepped weir, Weber number.

Resumen

Las fuertes lluvias estacionales sobre un embalse de la presa pueden provocar grandes inundaciones y la consiguiente falla de la estructura de la presa. Una forma de controlar los efectos adversos de las lluvias torrenciales es usar presas con dimensiones más grandes, como las presas de tecla de piano (PKW). A pesar de aumentar los costos generales del proyecto, estos vertederos son preferidos debido a su alta capacidad de alcantarilla. Los PKW pueden causar daños a la presa al crear chorros de agua abajo. Se ha hecho un esfuerzo en el presente estudio para examinar la disipación de energía debido a la instalación de escalones y deflectores con diferentes tamaños en vertederos de teclas de piano. Los resultados han mostrado que pequeños deflectores y grandes escalones causan pérdidas de energía máximas de 85 y 80% en los vertederos estudiados, respectivamente. También se ha descubierto que la disipación de energía del vertedero estaba inversamente relacionada con su ángulo de inclinación (el ángulo entre la salida del vertedero y la pared vertical de los escalones / deflectores). Los otros hallazgos son: (1) aumentar la velocidad de flujo redujo el efecto de los pasos en la pérdida de energía; (2) el número de Weber ha sido directamente proporcional a la pérdida de energía; y (3) la relación yc/H (profundidad crítica a la relación de la

cabeza del vertedero) ha sido directamente proporcional a la pérdida de energía.

Palabras clave: vertedero de tecla de piano, disipación de energía, vertedero escalonado, número de Weber.

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Introduction

Destructive human activities in recent years have resulted in severe climatic change as well as heavy rains over dam reservoirs (Beniston *et al.*, 2007). Severely heavy rains would rapidly increase water volume in the dam reservoir, eventually leading to the hydrodemolition of the dam. According to the research conducted by The United States Army Corps of Engineers (USACE), 36 percent of the existing dams are unsafe, with 80 percent of them being unsafe due to the low culvert capacity of their weirs (Suprpto, 2013). Due to the catastrophic risks this problem poses to human life, engineers have paid particular attention to increasing the culvert capacity of weirs, which is achieved in most cases by enlarging the dimensions of the weir (ASDSO, 2012). However, this solution is not

always effective due to such limitations as insufficient dam space and high costs. For this reason, researchers have been trying to increase the efficiency of dam weirs by modifying their structure (Tiwaria & Sharmam, 2015). The piano key weir (PKW) is one such solution (Figure 1). At a width equal to that of a linear weir, the PKW can produce a larger length and discharge coefficient product (CdL) at a lower Cd (Equation (1)), thus raising its hydraulic efficiency by 3-4 times as compared to the linear weir (Tiwaria & Sharmam, 2015):

$$Q = \frac{2}{3} C_d L \sqrt{2gH_t^3} \quad (1)$$

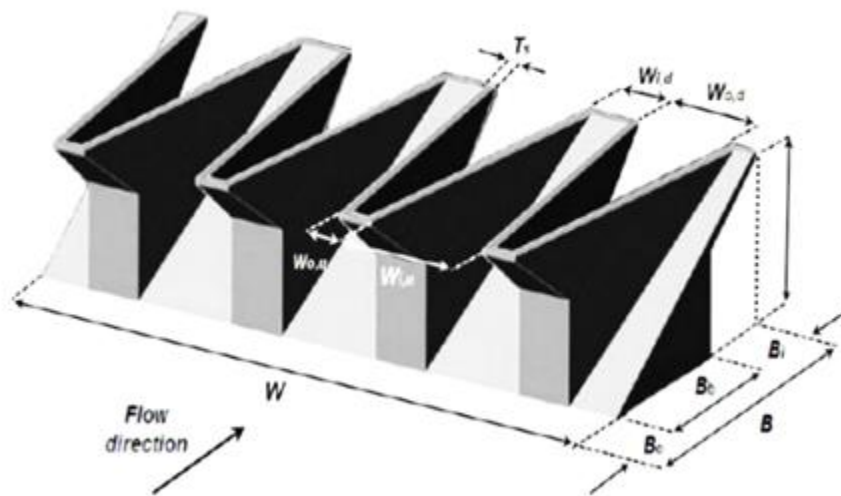


Figure 1. Main parameters in a piano key weir (PKW).

In Equation (1), Q is the flow rate; Cd is the discharge coefficient; L is the developed crest length, and H_t is the total weir head (sum of the

velocity head and the static head) (Tullis, Young, & Chandler, 2007). Due to this advantage, the PKW has received widespread attention among researchers in recent years, prompting them to conduct much research on this type of weir (Ribeiro, Pfister, & Schleiss, 2013) (Table 1).

Table 1. Lists the characteristics of the PKW.

| | |
|------------------|--|
| <i>BT</i> | Total length |
| <i>B</i> | Base length |
| <i>Bo</i> | Upstream(outlet key) overhang crest length |
| <i>Bi</i> | Downstream(inlet key) overhang crest length |
| <i>P</i> | Height of the inlet entrance measured from the PKW crest |
| <i>Wi</i> | Inlet key width (sidewall to sidewall) |
| <i>Wo</i> | Outlet key width (sidewall to sidewall) |

In a PKW, energy dissipation is caused by the increase of water head at the upstream section of the dam as well as the formation of vortex regions due to the flow impacting the weir walls (in the lower parts in particular) (Kabiri-Samani & Javaheri, 2012).

The PKW was first proposed by Lempérière and Ouamane (2003). In their study, they showed that there were two dominant flows in the PKW which is classified as a multi-purpose curved weir. In the first model, the inlet key attracts the nearby flows and subsequently discharges them (via precipitation) over the inlet crest to the downstream section (similar to a sharp-crested weir with a sloped body) (Lempérière, 2017). In the

second model, the flow is discharged into the downstream section of the key slope upon passing over the outlet crest (Lempérière, 2017). However, due to the jet damaging the downstream structure, measures including increasing wall thickness and building still basins at the downstream section are necessary to maintain the stability of this structure. In spite of their high initial costs, stepped weirs can reduce energy dissipation in the weir, thus reducing the overall construction costs (Chanson, 2001). The energy dissipation effect of the steps would reduce the depth, the length, and the lateral wall thickness of the downstream still basin, thus leading to substantially reduced construction costs (Chanson, 2001). In their study, Dermawan and Legono (2011) examined the residual energy and the relative energy dissipation in a model stepped weir (slope = 45°; height = 100 cm; width = 30 cm). In their test which comprised five steps, they placed 2, 4, 8, 16, and 32 steps between the weir crest and its toe, and measured five different flows (between 1.73 and 6.15 l/s) for each test step. Their obtained results showed that, no matter how many test steps were conducted, energy dissipation decreased with increasing the yc/H ratio.

In this study, the energy dissipation in a PKW was initially measured. Then, by mounting steps and baffles onto the weir, the respective energy dissipation in each case was determined.

Materials and methods

The experiments were conducted in a metallic flume (length = 7 m; width = 60 cm) with glass walls at the Water and Electricity Organization Laboratory, Khuzestan, Iran. A point depth meter (measuring accuracy = 1 mm) was used to measure water height at the upstream and downstream of the weir. The flow rate was measured via a manometer and a calibrated triangular weir (Figure 2, a).

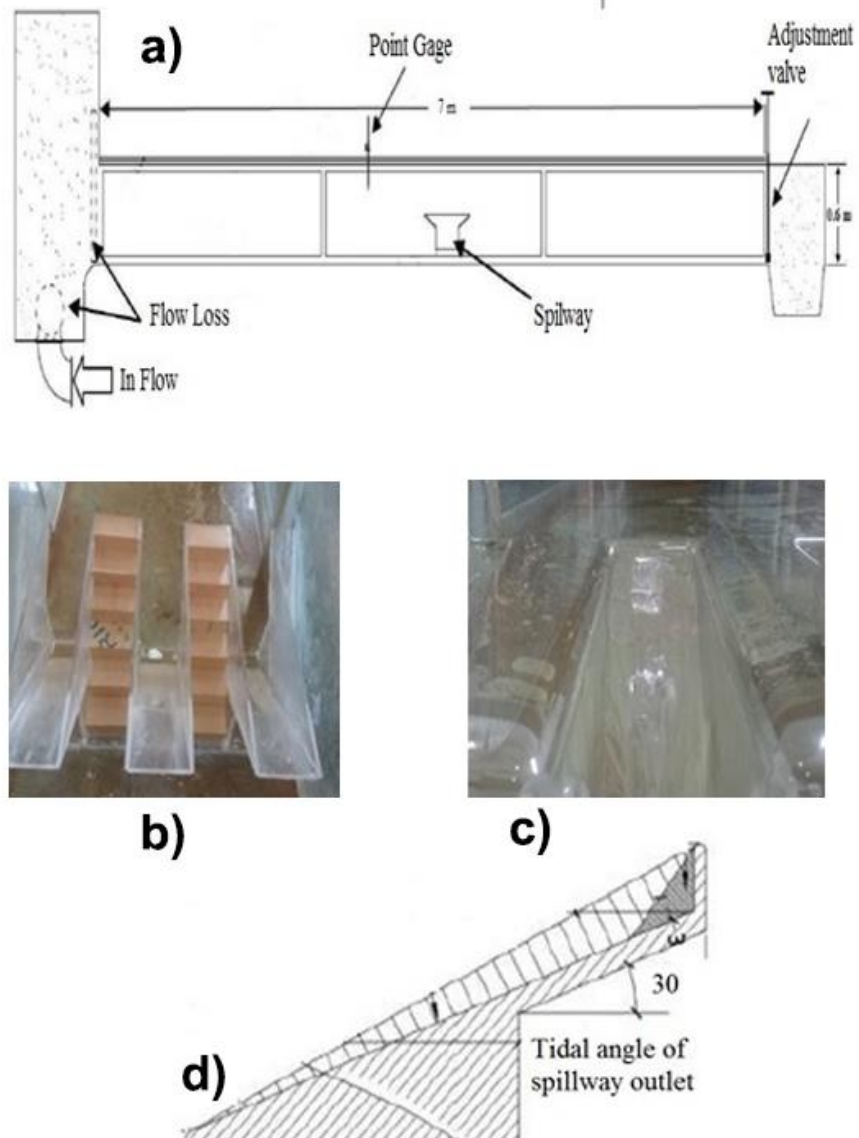


Figure 2. (a) Schematic of the laboratory flume; (b) stepped PKW; (c) baffled PKW, and (d) schematic of weir angle.

Three Plexiglas models with slope angles of 30, 35, and 40 were developed for simulating the PKW (Figure 2, b, c, d).

In this study, W_i is the inlet key width (= 10 cm); W_0 is the outlet key (= 10 cm); W_u is the width of one cycle; W is the flume width; B is the weir length (from upstream to downstream) (= 40 cm); B_i is the length of the downstream slope; H_r is the tailwater (downstream) depth of the weir; H_s is water height over the crest of the weir; P is the weir height (= 20 cm); L is the weir crest length (= 40 cm), and L_0 is the length of the outlet key

Separate parts were used to simulate the steps and baffles (H_b represents the step/baffle height). Upon installing the weir and sealing it with aquarium glue, a butterfly valve was used to control the water flow rate into the flume. Water depths at the weir upstream (in the laminar region 40 cm from the weir, for higher accuracy) and at the downstream of the key outlet were measured via the depth meter. Equation (2) and Equation (3) were used to calculate the energy dissipation in the weir:

$$E_0 = y_0 + \frac{V_0^2}{2g} \quad (2)$$

$$E_1 = y_1 + \frac{V_1^2}{2g} \quad (3)$$

Using the upstream water depth (y_0), the downstream water depth (y_1), and the flow velocity (calculated from Equation (4)), the energy at each section was calculated. The relative energy dissipation for the flow was then obtained from the following equation:

$$\Delta E_r = \frac{\Delta E}{E_0} = \frac{E_0 - E_1}{E_1} \quad (4)$$

Dimensional analysis

The parameters affecting the energy drop are shown in Equation (5):

$$F = (W_i, W_0, W_u, W, B, B_i, H_r, H_s, y_c, g, \rho, \sigma, H, L_0, P, L, \mu, Q) \quad (5)$$

where y_c represents the critical depth ($y_c = (q^2/g)^{1/3}$); H is the height of water on the weir; σ is the surface tension (for water at 20° C: 0.074); ρ is the mass of a unit volume of water (998.2 kg/m³); μ is the dynamic viscosity; q is the specific discharge ($q = \frac{Q}{W}$), and g is the gravitational acceleration (9.8 m/s²). The following parameters were obtained by conducting dimensional analysis using Buckingham π theorem:

$$f = \left(\frac{H_d}{P}, \frac{H_s}{P}, \frac{H_s}{D}, \frac{Y_c}{H}, \frac{\Delta E}{H}, \frac{W_i}{H}, \frac{L}{H}, \frac{W}{H}, \frac{P}{H}, \frac{B_i}{H}, \frac{H_t}{P}, \frac{B_o}{H}, \frac{R}{H}, \frac{E_0}{H}, N, \frac{Q}{\sqrt{2g}LH^{1.5}}, \frac{\sigma}{\rho V^2 H}, \frac{\rho V H}{\mu}, \frac{V}{\sqrt{gH}} \right) = 0 \quad (6)$$

In Equation (6), S is the ratio of R/B' , and the ratios of $\frac{Q}{\sqrt{2g}LH^{1.5}}, \frac{\sigma}{\rho V^2 H}, \frac{\rho V H}{\mu}, \frac{V}{\sqrt{gH}}$ are Froude number (Fr), Weber number (We), and

discharge coefficient (C_d), respectively. In Equation (5) and Equation (6), W_i is the width of the inlet; W_o the width of the outlet; W_u the cycle width; W the flume width; B , the upstream-downstream length of weir; B_i , the downstream length of the overhang; H_r , the water depth at the base of the weir; H_s , the water level at the top of the weir; Y_c , the critical depth; H , the water level over the weir; σ , surface tensión; ρ , water density; μ , dynamic viscosity; L , the weir crest length; L_o outlet length; g , gravitational acceleration, and Q is the discharge coefficient.

Since the lowest Reynolds number in this paper is 3 500, the effect of Reynolds number is neglected. Moreover, given the fixed weir geometry, the final relation for the dimensionless parameters affecting the coefficient of discharge is expressed by Equation (7):

$$C_d = f\left(\frac{H_d}{P} \cdot \frac{Y_c}{H} \cdot W_e \cdot Fr\right) \quad (7)$$

It should be noted that the Froude number at the upstream of weir for all the models were below 1, meeting the conditions subcritical flow at the upstream.

Results and discution

To study the effect of the steps and baffles on the energy dissipation in the weir, different experiments were designed where models with different step heights (1, 2, and 3 centimeters); three cubic baffles (with side lengths of 1, 2, and 3 centimeters), and three PKWs (with slope angles of 30, 35, and 40 degrees) were tested. Overall, 180 experiments were conducted at flow rates ranging from 1 to 50 l/s. The relative energy dissipation ($\frac{\Delta E}{E_0}$) was obtained from Equation (8):

$$\Delta E_r = \frac{\Delta E}{E_0} = \frac{E_0 - E_1}{E_1} \quad (8)$$

The respective effects of the dimensionless yc/H ratio (which expresses the effect of flow rate as well as weir water head) and the Weber number (which indicates the effect of surface tension due to direct contact between the flow and the weir structure) on the energy dissipation were duly examined. Table 2 shows the summary of the results.

Table 2. The summary of the results.

| Model | The angle | Max relative energy loss | yc/H |
|---------------|------------------|---------------------------------|--------------------------|
| Observational | 30 | 8.2125 | 1.69 |
| Bafel 3 cm | 30 | 8.5565 | 1.93 |
| Bafel 2 cm | 30 | 8.5565 | 1.93 |
| Bafel 1 cm | 30 | 8.2125 | 1.69 |

| | | | |
|---------------|----|--------|------|
| Observational | 35 | 8.6487 | 1.5 |
| Bafel 3 cm | 35 | 8.1682 | 1.93 |
| Bafel 2 cm | 35 | 1.9412 | 1.13 |
| Bafel 1 cm | 35 | 6.1749 | 1.69 |
| Observational | 40 | 2.9417 | 1.04 |
| Bafel 3 cm | 40 | 1.9551 | 1.04 |
| Bafel 2 cm | 40 | 1.9551 | 1.04 |
| Bafel 1 cm | 40 | 1.6857 | 1.04 |
| Observational | 30 | 8.2125 | 1.69 |
| Stairs 3 cm | 30 | 7.2535 | 1.69 |
| Stairs 2 cm | 30 | 7.7988 | 1.5 |
| Stairs 1 cm | 30 | 6.8365 | 1.52 |
| Observational | 35 | 8.6487 | 1.5 |
| Stairs 3 cm | 35 | 6.2094 | 1.5 |
| Stairs 2 cm | 35 | 7.0511 | 1.59 |
| Stairs 1 cm | 35 | 6.3829 | 1.57 |
| Observational | 40 | 2.9417 | 1.04 |
| Stairs 3 cm | 40 | 3.5269 | 1.39 |
| Stairs 2 cm | 40 | 3.407 | 4.71 |
| Stairs 1 cm | 40 | 3.8271 | 1.34 |

Effect of yc/H on relative energy loss

The effect of yc/H on the relative energy dissipation was studied in two parts. In the first part (depicted in Figure 3), the effect of step/baffle height on energy loss; and in the second part (shown in Figure 4), the effect of slope angle on energy dissipation was studied.

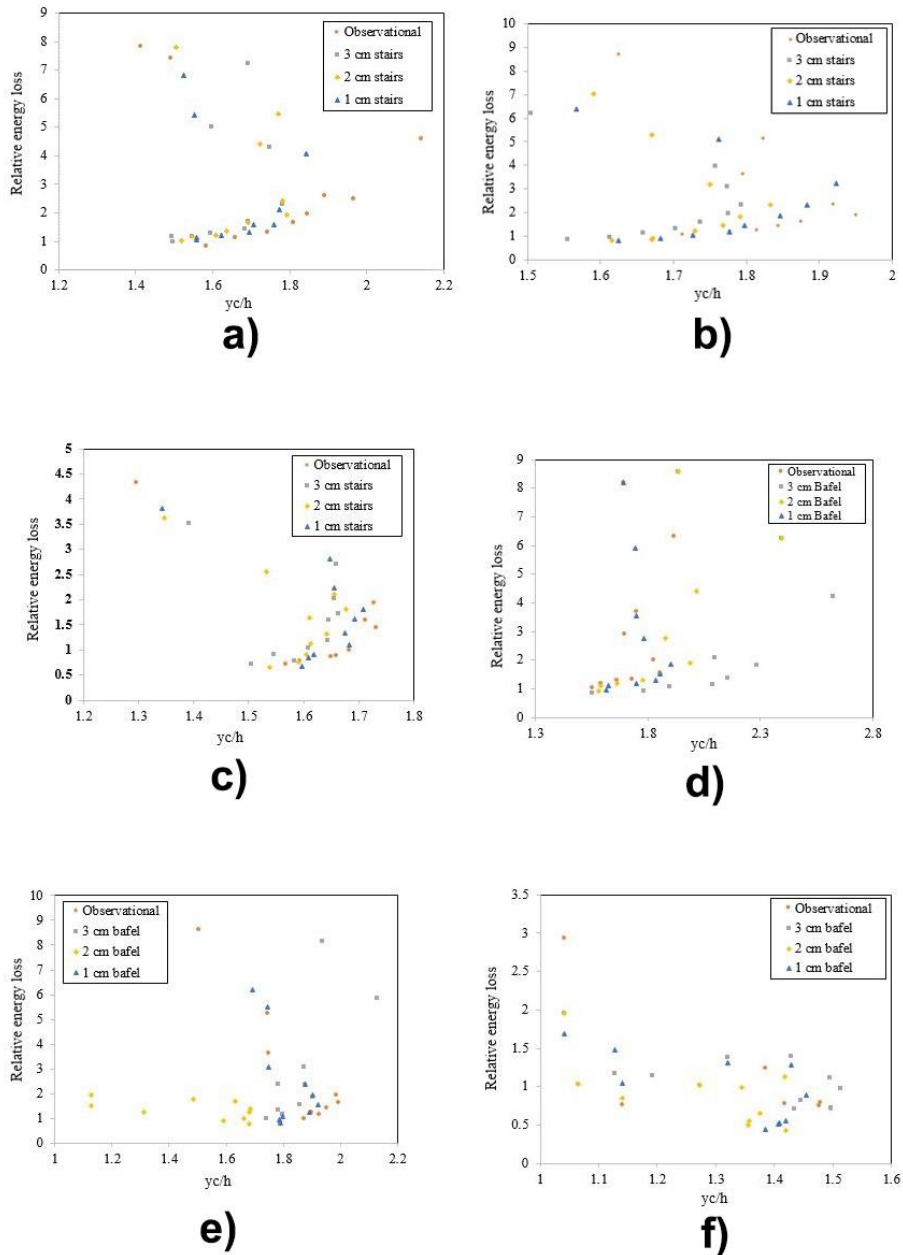


Figure 3. Stepped weir with (a) 30° slope angle; (b) 35° slope angle; (c) 40° slope angle and baffled weir with (d) slope angle of 30; (e) slope angle of 35°; (f) slope angle of 40°.

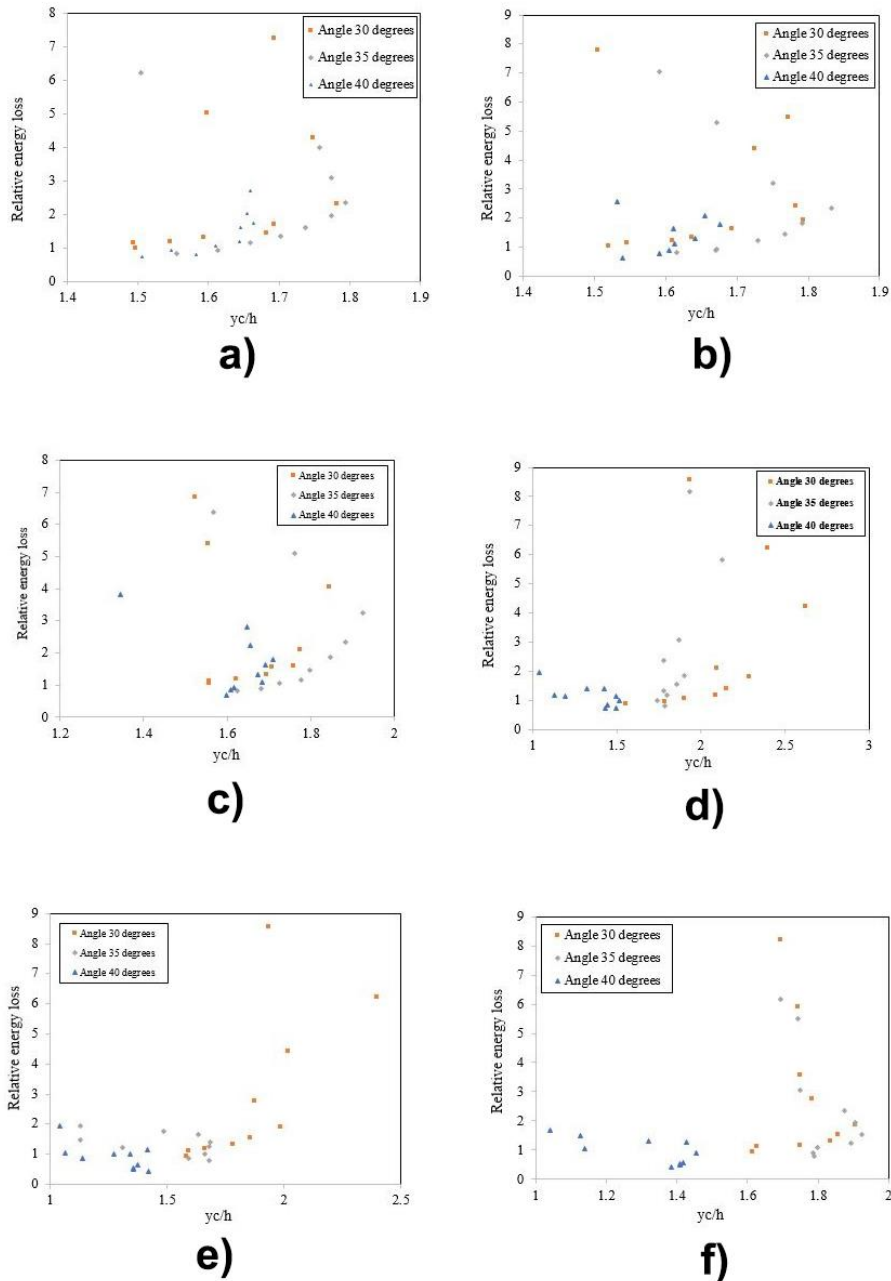


Figure 4. (a) A large step (height = 3 cm); (b) a medium step (height = 2 cm); (c) a small step (height = 1 cm); (d) a large baffle (height = 3

cm); (e) a medium baffle (height = 2 cm); (f) a small baffle (height = 1 cm).

As can be observed in Figure 3 (a, b, c) (where energy losses corresponding to three different slope angles were calculated), increasing yc/H from 1.5 to 1.7 increased the relative energy loss. However, this trend was reversed as yc/H was increased from 1.3 to 1.4, which led to a corresponding decrease in the energy loss. According to these diagrams, increasing the step height increased energy loss. However, the greatest energy dissipation in the unstepped weir occurred at $yc/H > 1.3$. Since yc/H exhibits the effect of flow rate, and this increase in energy dissipation occurred at smaller yc/H values, we can conclude that, at lower flow rates (*i.e.*, lower flow velocities), a smaller flow separation area was formed in the unstepped weir, thus causing greater energy loss.

As can be observed in Figure 3 (d, e, f), increasing dimensions of the cubic baffles decreased energy dissipation at all the three slope angles. This result was enhanced as the flow rate increased since, at lower flow rates, the energy dissipation difference among the three models was very small. The greatest energy dissipation was obtained for the weir with a 30° slope angle fitted with a small baffle. This result confirms the fact that reducing the outlet key space would increase the flow rate as well as the energy in a PKW. Hence, smaller baffles must be used for obtaining greater energy loss.

Figure 4 (a, b, c) shows the effect of the slope angle on the energy dissipation in the tested stepped weirs. It was concluded from these

diagrams that the greatest energy dissipation corresponding to the $1.6 < yc / H < 2.0$ interval was obtained in weirs with greater step heights. However, the maximum dissipation in the lower yc/H values occurred at a step height of 1 cm; indicating that, at higher velocities, the energy dissipation increased in weirs with a step height of 3 cm due to the formation of a smaller separation zone in such weirs.

Figure 4 (d, e, f) shows the effect of weir slope angle on energy dissipation in weirs with cubic baffles. As can be observed, energy dissipation is inversely related to slope angle, so that increasing the former would lead to a decrease in the latter. Therefore, the effect of baffles on energy dissipation reduction diminished as baffle height was increased.

Effect of Weber number on relative energy dissipation

The Weber number was calculated from Equation (9):

$$We = \frac{\rho V^2 H_s}{\sigma} \quad (9)$$

Figure 5 and Figure 6 show the results obtained in the section. As in the previous section, these results entail two parts.

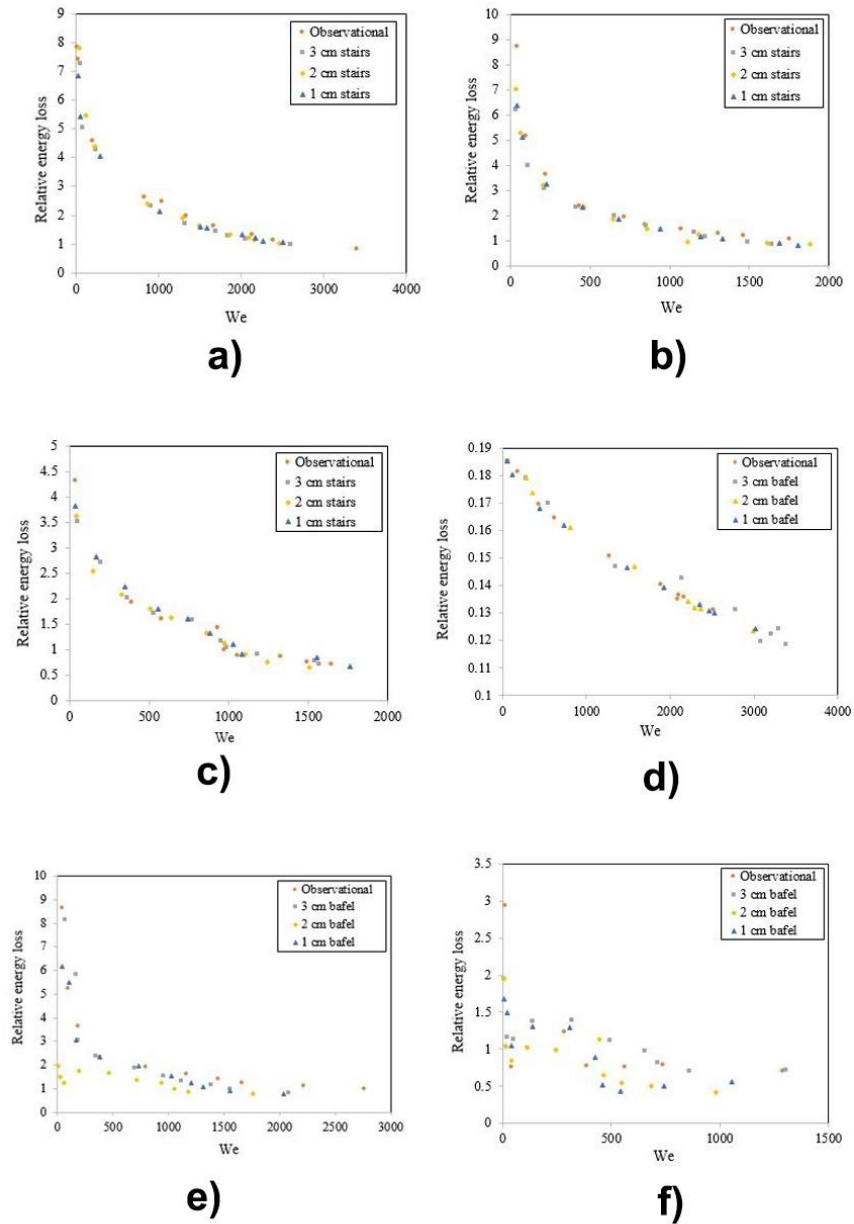


Figure 5. Stepped weir with (a) a slope angle of 30°; (b) a slope angle of 35°; (c) a slope angle of 40°; (d) a 30° baffle; (e) a 35° baffle; (f) a 40° baffle.

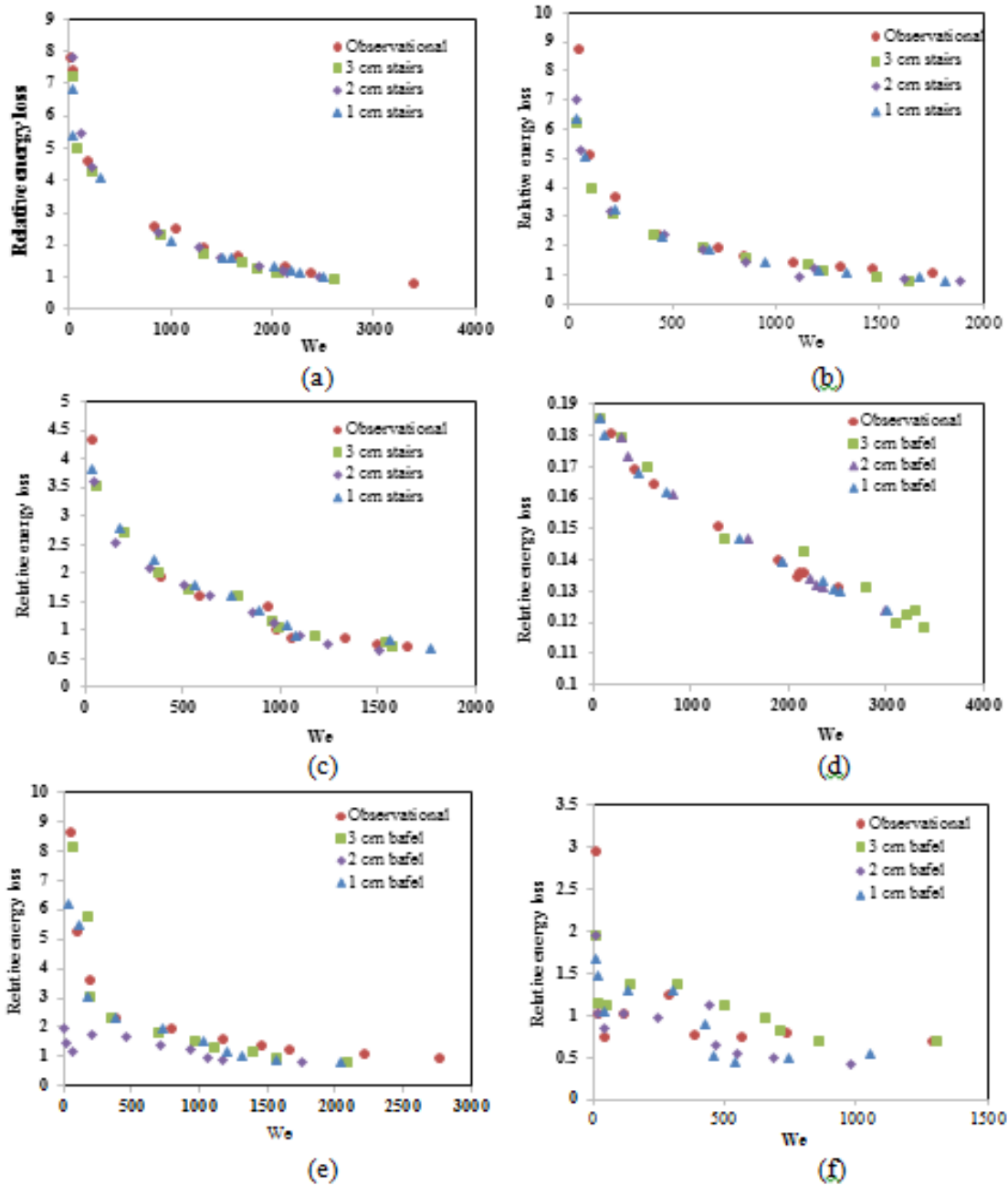


Figure 6. (a) A large-sized step (height = 3 cm); (b) a medium-sized step (height = 2 cm); (c) a small-sized step (height = 1 cm); (d) a

large-sized baffle (height = 3 cm); (e) a medium-sized baffle (height = 2 cm); (f) a small-sized baffle (height = 1 cm).

As can be observed in Figure 5 (a, b, c), increasing the Weber number reduced the energy loss. In the $1 < We < 500$ range, a 45% energy dissipation was obtained for the unstepped weir. Increasing the Weber number changed this trend however, leading to greater energy dissipation in the stepped weirs. A maximum energy dissipation of 5% was obtained at $We = 500$ in the weir with a step height of 3 cm. As can be clearly observed in these figures, the greatest relative energy losses occurred at $We > 1\ 500$ for the following weirs respectively: the weir with no steps, the weir with small-sized steps, the weir with medium-sized steps, and the weir with large-sized steps. As can be observed in Figure 5 (d, e, f), increasing the Weber number led to a decrease in energy loss. Conversely, the greatest energy losses were obtained for the weirs with large baffles, medium baffles, small baffles, and the weirs without baffles, respectively.

As can be observed in Figure 6 (a, b, c), as the Weber number increased, the greatest energy losses of circa 69, 65, and 39 percent were obtained for the weirs with steps heights of 1, 2, and 3 centimeters, respectively.

As can be observed in Figure 6 (d, e, f), similarly to the stepped weirs, the energy dissipation in the baffled weirs as well as the effects of the installed steps and baffles decreased with the slope angle. However, unlike the case of stepped weirs where increasing the step height reduced

the relative energy dissipation, increasing the height of the cubic baffles led to a corresponding reduction in energy dissipation

Effect of specific discharge coefficient

In this section, the effect of step height on the specific discharge coefficient (which is an index of weir efficiency) is examined (Figure 7).

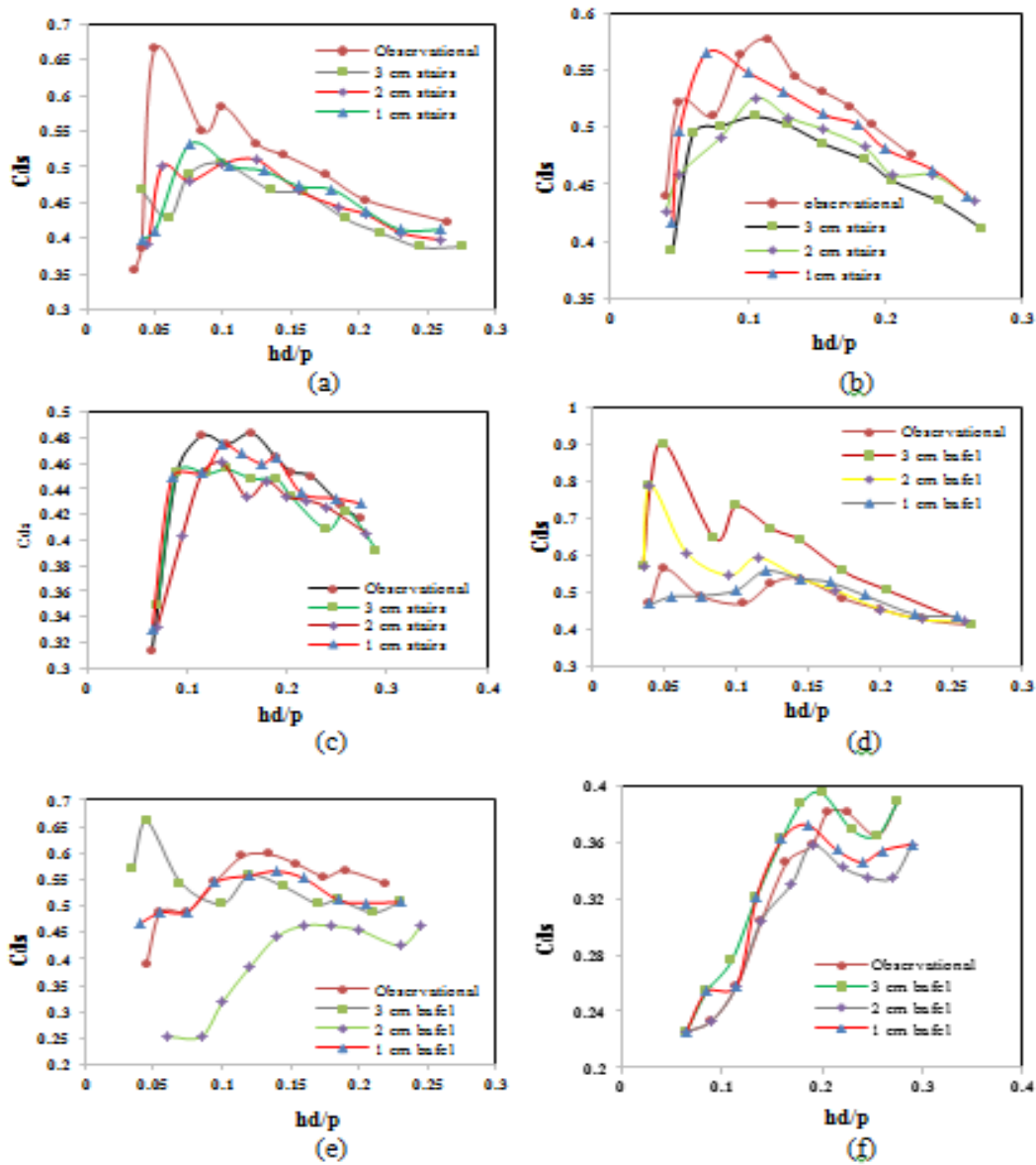


Figure 7. Variations of C_{ds} with hd/p at (a) slope angle of 30°; (b) slope angle of 35°; (c) slope angle of 40°; (d) slope angle of 30°; (e) slope angle of 35°; (f) slope angle of 40°.

As can be observed in the Figure 7, the maximum discharge coefficient was obtained for the unstopped weir which yielded the highest efficiency; and increasing the step height reduced the specific discharge coefficient. It can be concluded from Figure 7 that a larger baffle would increase weir efficiency beyond the observed value. This exhibits the fact that though a large baffle exhibits a negative effect in terms of energy loss, it brings about a positive effect in terms of efficiency by increasing the weir efficiency.

As can be observed in Figure 8, the maximum discharge coefficient (0.65) was obtained for the unstepped weir which also exhibited the highest efficiency. Increasing step height also led to a corresponding decrease in the specific discharge coefficient. A maximum discharge coefficient of 0.55 was obtained for the 35° weir with a step height of 1 cm, whereas a minimum discharge coefficient of 0.34 was obtained in all the three weir models for a step height of 3 cm.

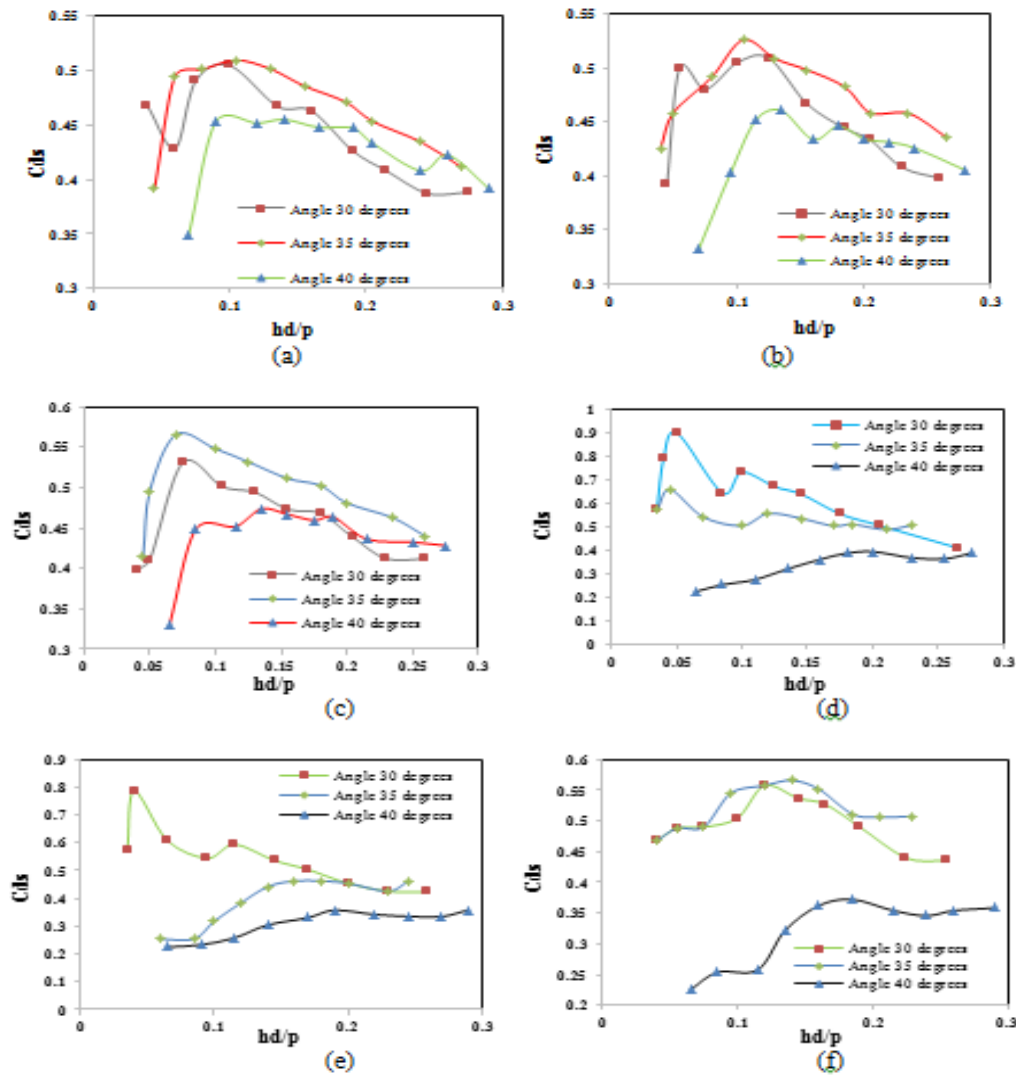


Figure 8. Variations of C_{ds} with hd/p at (a) step height of 3 cm; (b) step height of 2 cm; (c) step height of 1 cm; (d) a 3 cm baffle; (e) for a 2 cm baffle; (f) a 1 cm baffle.

It can be concluded from Figure 7 and Figure 8 that increasing the slope angle led to a corresponding increase in weir efficiency. However, increasing the baffle dimensions decreased the efficiency: for the 3 cm

baffle, a maximum efficiency of 0.85 was obtained at an angle of 30°. For the 2 cm (medium) baffle, a maximum energy dissipation of 0.8 was measured at an angle of 30°. Ultimately, a maximum efficiency of 0.55 was obtained for the 1 cm baffle at the same angle of 30°.

Conclusions

The results obtained in the present study showed that baffled and stepped weirs produced energy losses of 85 and 80%, respectively. Therefore, baffles have a greater effect on the energy dissipation across the weir. In addition, the slope angle of the outlet keys was found to be inversely related to the weir energy loss. However, the most significant point was that the effect of weir steps on energy dissipation decreased with increasing flow rate.

At Weber numbers lower than 1 500, the greatest energy losses were obtained for the weir with no steps, the weir with small steps, the weir with medium steps, and the weir with large steps respectively. As for the baffled weirs, the greatest losses were achieved for the weir with large baffles, the weir with medium baffles, the weir with small baffles, and the weir with no baffles respectively.

Increasing the height of steps led to a corresponding increase in the energy dissipation in the stepped weirs. In the baffled weirs, however,

the reverse occurred, *i.e.*, the energy dissipation increased by reducing the baffle height.

The greatest discharge coefficient (0.65) was obtained for the unstepped weir which exhibited the highest efficiency. Increasing the step height reduced the specific discharge coefficient. A larger efficiency (than the observed efficiency) was obtained by installing a large baffle on the weir; however, decreasing the baffle dimension led to a corresponding energy drop resulting in a reduction in efficiency below the observed efficiency.

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References

- ASDSO, Association of State Dam Safety Officials. (2012). *Summary of Existing Guidelines for Hydrologic Safety of Dams* (pp. 1-22). Lexington, USA: Federal Emergency Management Agency.
- Beniston, M., Stephenson, D., Christensen, O., Ferro, C., Frei, C., Goyette, S., & Koffi, B. (2007). Future extreme events in European Climate: An exploration of regional climate model projections. *Climatic Change*, 81(1), 71-95. DOI: 10.1007/s10584-006-9226-z
- Chanson, H. (2001). *The hydraulics of stepped chutes and spillways*. Lisse, The Netherlands: Balkema.

- Dermawan, V., & Legono, D. (2011). Residual energy and relative energy dissipation on stepped spillway. *Journal of Applied Technology in Environmental Sanitation*, 1(4), 381-392.
- Kabiri-Samani, A., & Javaheri, A. (2012). Discharge coefficients for free and submerged flow over piano key weirs. *Journal of Hydraulic Research*, 50(1), 114-120. DOI: 10.1080/00221686.2011.647888
- Lempérière, F. (2017). Dams and floods. *Engineering*, 3(1), 144-149.
- Lempérière, F., & Ouamane, A. (2003). The piano keys weir: A new cost-effective solution for spillways. *International Journal of Hydropower & Dams*, 5, 144-149.
- Ribeiro, M. L., Pfister, M., & Schleiss, A. J. (November, 2013). Overview of piano key weir prototypes and scientific model investigations. In: *Proceedings of the 2nd International Workshop on Labyrinth and Piano Key Weirs*, Paris, France.
- Suprpto, M. (2013). Increase spillway capacity using labyrinth weir. *Procedia Engineering*, 54, 440-446. DOI: 10.1016/j.proeng.2013.03.039
- Tiwaria, H., & Sharma, N. (2015). Developments to improve hydraulic competence of spillways. *Aquatic Procedia*, 4, 841-846.
- Tullis, B. P., Young, J. C., & Chandler, M. A. (2007). Head-discharge relationships for submerged labyrinth weirs. *Journal of Hydraulic Engineering*, 133(3), 248-254. DOI: 10.1061/(ASCE)0733-9429(2007)133:3(248)