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Notes

Correction of design floods due to hydrological uncertainty

Corrección de las crecientes de diseño por incertidumbre hidrológica

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Abstract

The planning, design and/or hydrological inspection of hydraulic works is based on what are called *Design Floods* (DF) or maximum river flows associated with low probabilities of exceedance. DFs are estimated using Frequency Analysis (FA), a statistical method that ignores the hydrological uncertainties involved. When such indecisions are taken into account, DFs are not unique magnitudes but a range of values, and therefore, the design and inspection of hydraulic infrastructure becomes an indeterminate process. DFs can be corrected, according to hydrological uncertainty, using a simple practical procedure developed by Botto, Ganora, Claps and Laio (2017). The method is based on a correction factor ($\hat{y}$), which modifies the DFs obtained with FA. The value of $\hat{y}$ is obtained with several empirical equations that are a function of the size of the available records of annual maximum flows and of the return period in years or the inverse of the exceedance probability. There are five equations of $\hat{y}$, one for each of the most used probability distribution functions (PDF) in the FA, these are: Log–Pearson type III, General of Extreme Values, Generalized Logistic, Log–Normal and Pearson type III. The corrective method was applied to seven flood registers, after objective selection of the most appropriate PDF, according to their L moment ratios. We obtained that the corrective factors ($\hat{y}$) vary in the 50-year *Tr* from 1% to 5.6%, while in the 1000-year *Tr* corrections fluctuated between 14.9% and 43.9%.

Keywords: frequency analysis, design floods, hydrological uncertainty correction, L moments, L moment ratios, LP3, GEV, LOG, LN3 and PT3 distributions, minimum absolute distance, standard error of fit.

Resumen

La planeación, diseño y/o revisión hidrológica de las obras hidráulicas se basa en las llamadas crecientes de diseño (CD) o gastos máximos del río asociados con bajas probabilidades de excedencia. Las CD se estiman por medio del análisis de frecuencia (AF), método estadístico que ignora las incertidumbres hidrológicas involucradas. Cuando tales indecisiones son tomadas en cuenta, las CD no son magnitudes únicas, sino una gama de

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valores, y entonces, el diseño y la revisión de la infraestructura hidráulica se vuelve un proceso indeterminado. La corrección de las CD conforme a la incertidumbre hidrológica se puede realizar de forma práctica y simple, con base en el procedimiento que han desarrollado Botto, Ganora, Claps y Laio (2017), fundamentado en un factor de corrección ($\hat{y}$), que modifica las CD obtenidas con el AF. El valor de $\hat{y}$ resulta de varias ecuaciones empíricas, función de tamaño del registro disponible de gastos máximos anuales y del periodo de retorno (*Tr*) en años o inverso de la probabilidad de excedencia. Se dispone de cinco ecuaciones de $\hat{y}$, una para cada función de distribución de probabilidades (FDP) más utilizada en el AF, éstas son: Log-Pearson tipo III, general de valores extremos, logística generalizada, Log-Normal y Pearson tipo III. El método correctivo se aplicó a siete registros de crecientes, previa selección objetiva de la FDP más adecuada, según sus cocientes de momentos L. Se obtuvo que los factores correctivos ($\hat{y}$) varían en el *Tr* de 50 años del 1 al 5.6%; en cambio, en el *Tr* de 1 000 años, las correcciones fluctuaron del 14.9 hasta el 43.9%.

Palabras clave: análisis de frecuencias, crecientes de diseño, corrección de incertidumbre hidrológica, momentos L, cocientes de momentos L, distribuciones LP3, GVE, LOG, LN3 y PT3, distancia absoluta mínima, error estándar de ajuste.

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Introduction

Flood frequency analysis

Planning, design, construction, and inspection of hydraulic works, such as reservoirs, protective dikes, canals, bridges and urban drainage, are based on what are called *Design Floods* (DF), which are maximum flows of a river associated with low exceedance probabilities. The most reliable DF are estimated with the *Flood Frequency Analysis* (FFA).

Around the middle of the first decade of the 20th century probabilistic concepts were first applied to the problem of DF estimation. The objective was to replace techniques based on enveloping curves and empirical formulas with a more objective and precise method. By mid-20th century, the existence of broad registers of annual maximum flows and theoretical advances that developed new probabilistic models allowed arrival at a standard approach for FFA (Merz & Blöschl, 2008).

FFA consists basically of fitting a *probability distribution function* (PDF) to the ordered sequence of annual maximum flows to extrapolate its right tail and make *predictions* or estimations with low exceedance probabilities. This process comprises the following four steps: (1)

verifying the randomness of the flood registers to be processed; (2) fitting several PDF using the moments, maximum likelihood and L moments methods; (3) selecting the best fit using quantitative measures, such as standard errors of fit and absolute mean; and (4) obtaining the relevant predictions (Kite, 1977; Stedinger, Vogel, & Foufoula–Georgiou, 1993; Rao & Hamed, 2000; Meylan, Favre, & Musy, 2012; Stedinger, 2017).

FFA involves several weaknesses or hydrological uncertainties that can be specified in three aspects. The first refers to the representativity of the available register of floods in the future, which can be altered by physical changes in the basin or by regional or global climate change. Also implied is that all the registers can contain errors in measurement or sampling. The second generator of uncertainty is associated with extrapolation that must be done to estimate return periods of 100, 500 and 1000 years because, generally, the available registers of floods do not exceed 80 years. Finally, the third source of uncertainty lies in the method itself because one PDF must be selected in order to make the predictions (Merz & Blöschl, 2008).

UNCODE Procedure

Botto, Ganora, Laio and Claps (2014) proposed the procedure called *Uncertainty Compliant Design Flood Estimation*, or UNCODE. This is a novel approach that parts from the interval, or range, of possible values for each DF and converges in a single design. Using UNCODE, DFs can be estimated in accord with hydrological uncertainty since it permits selecting significant flood estimations from its PDF that exist in each return period value (*Tr*); when additional restrictions are posed based on the cost–benefit criterion and are resolved numerically by means of a simulation scheme.

The criterion minimum cost–benefit can be consulted in Campos-Aranda (2015). Botto, Ganora, Claps and Laio (2017) present a summary of the UNCODE operative process, and in Kjeldsen, Lamb and Blazkova (2014) general aspects relative to uncertainty in the FFA can be consulted, and more specific aspects in Burn (2003), and Cheng, Chang and Hsu (2007).

Objective

Botto *et al.* (2017) were able to reach concrete results of UNCODE using a quite simple procedure that corrects DF estimated with FFA using a correction factor, register size function (*n*), and the return period (*Tr*). The result is an estimation of the DF considering hydrological uncertainty ($\hat{Q}^{*}_{Tr}$). The fundamental objective of this Note is to make this procedure known, highlighting the operative part of the L moments and an ideal selection of PDF based on the minimum absolute distance calculated with L moment quotients. This is done in seven real numerical applications in registers of annual maximum flow in Hydrological Region Num. 10 (Sinaloa), Mexico.

Methods and materials

Estimation of the correction factor

Botto *et al.* (2014) have demonstrated that the DF obtained by applying UNCODE (Q^*_{Tr}) is always a value greater than the value estimated (Q_{Tr}) with FFA, which is called *uncertainty-free*. This increase reaches 55% in long *Tr* and small *n*. Moreover, Botto *et al*. (2017) have defined that the relative difference between the two values corresponds to the *correction factor* (y), that is:

$$y = \frac{Q^*_{Tr} - Q_{Tr}}{Q_{Tr}} \qquad (1)$$

According to Botto *et al.* (2017), y increases with *Tr* due to the increase in Q_{Tr} estimation uncertainty and, for a fixed *Tr*, it also grows with variability in the probability distribution that allows estimating Q_{Tr}, which can be accepted to be in function of size (n) of the available register of annual maximum flows used in the FFA. From equation 1, we can obtain the estimation of $\hat{Q}^*_{Tr}$, as:

$$\hat{Q}^*_{Tr} = (1 + \hat{y}) \cdot Q_{Tr} \quad (2)$$

The best $\hat{y}$ estimator of the correction factor is obtained with the following expression, according to the results of Botto *et al.* (2017):

$$\hat{y} = \frac{1}{100} \exp\left[a_0 + a_1\sqrt{n} + a_2 \cdot \ln(Tr)\right] \quad (3)$$

The coefficients a_0, a_1 and a_2, cited in Table 1, depend on the probability distribution used in the FFA and were evaluated by applying UNCODE extensively. Botto *et al*. (2017) used five *parent distributions*, eight sizes (n) from 30 to 100 in intervals of ten, and 100 synthetic sequences for each combination of source model and size n. The parent distributions were: Log–Pearson type III (LP3), General of Extreme Values (GEV), Generalized Logistic (LOG), Log–Normal (LN3) and Pearson type III (PT3).

Table 1. Coefficients of Equation (3) and its adimensional statistical properties: coefficient of determination (R^2), mean absolute error (*MAE*) and mean standard error (*MSE*).

Parent distribution	a_0	a_0	a_0	R^2	*MAE*	*MSE*
LP3	0.78	−0.26	0.687	0.89	0.0235	0.0363
GVE	−2.27	−0.30	1.110	0.85	0.0190	0.0321
LOG	−2.36	−0.25	0.994	0.85	0.0096	0.0145
LN3	−0.82	−0.25	0.809	0.94	0.0107	0.0160
PT3	0.59	−0.24	0.567	0.96	0.0080	0.0115

The parent distributions of Table 1 are cited in chronological sequence, the order in which they were accepted as PDF of the application under precept. For this reason, the LP3 distribution was presented first, suggested in the late 1960s in the USA, then the GEV proposed in England in 1975, and finally, LOG, which substituted GEV as of 1999. LN3, of generalized application, was then cited in the FFA in the 1970s, and finally, the little used PT3 in FFA.

Botto *et al.* (2017) indicated that in 90% of the synthetic registers, their statistical properties oscillated in the following intervals for L moment ratios: variation $0.28 \leq \tau_2 \leq 0.40$, asymmetry $0.14 \leq \tau_3 \leq 0.40$, and kurtosis $0.07 \leq \tau_4 \leq 0.32$. Q_{Tr} values from FFA and their Q^*_{Tr} estimator of the exact value obtained with UNCODE were calculated for each of the 800 synthetic registers generated for each parent distribution. This step was carried out by adopting a distribution suitable to each synthetic series selected from among the five source models (LP3, GEV, LOG, LN3 and PT3). This reproduces the real FFA situation when the parent distribution is not known *a priori*. In this way, the error from the incorrect specification of probabilistic distribution is included in the results. The processed return periods (*Tr*) were the following five: 50, 100, 200, 500 and 1000 years.

In general terms, the obtained regressions and those proposed by Botto *et al*. (2017) are reliable, their coefficient of determination varying 0.85 to 0.96. Moreover, its analysis of residues by *MAE* y *MSE* in Table 1, define an overall mean value of 0.0181, that is, in the order of a 2% variation in $\hat{y}$ in estimation of a DF, for practical purposes is negligible.

L moments and quotients

L moments (λ_i) are linear combinations of the probability weighted moments (β_i), developed by Greenwood, Landwehr, Matalas and Wallis (1979); they are statistical parameters associated with ordered data. *L* moments are an efficient and robust system for fitting PDF currently in use or established under precept. The equations for calculating them are (Stedinger *et al.*, 1993; Hosking & Wallis, 1997; Rao & Hamed, 2000; Stedinger, 2017):

$$\lambda_1 = \beta_0 \quad (4)$$

$$\lambda_2 = 2 \cdot \beta_1 - \beta_0 \quad (5)$$

$$\lambda_3 = 6 \cdot \beta_2 - 6 \cdot \beta_1 + \beta_0 \tag{6}$$

$$\lambda_4 = 20 \cdot \beta_3 - 30 \cdot \beta_2 + 12 \cdot \beta_1 - \beta_0 \tag{7}$$

Moreover, *L* moment ratios (τ) are defined, beginning with *L–Cv*, which is analogous to this coefficient and later to those of similarity with the coefficients of asymmetry and kurtosis; these are:

$$\tau_2 = \lambda_2/\lambda_1 \tag{8}$$

$$\tau_3 = \lambda_3/\lambda_2 \tag{9}$$

$$\tau_4 = \lambda_4/\lambda_2 \tag{10}$$

In a sample size *n*, with its x_i elements arranged in ascending order ($x_1 \leq x_2 \leq \cdots \leq x_n$), the unbiased estimators of β_r are obtained with the following general expression:

$$\beta_r = \frac{1}{n}\sum_{j=r+1}^{n} \frac{(j-1)\cdot(j-2)\cdots(j-r)}{(n-1)\cdot(n-2)\cdots(n-r)} x_j \tag{11}$$

L quotient diagram

Has τ_3 in the abscissa axis and τ_4 in the ordinate axis. The three fitting parameter PDF are curved lines with the following polynomial-type equations (Hosking & Wallis, 1997):

Generalized logistic (LOG):

$$\tau_4^{\mathrm{LOG}} = 0.16667 + 0.83333 \cdot \tau_3^2 \quad (12)$$

Generalized Pareto (PAG):

$$\tau_4^{\mathrm{PAG}} = 0.20196 \cdot \tau_3 + 0.95924 \cdot \tau_3^2 - 0.20096 \cdot \tau_3^3 + 0.04061 \cdot \quad (13)$$

og–Normal (LN3):

$$\tau_4^{\mathrm{LN3}} = 0.12282 + 0.77518 \cdot \tau_3^2 + 0.12279 \cdot \tau_3^4 - 0.13638 \cdot \tau_3^6 + 0.11368 \cdot \tau_3^8 \quad (14)$$

Pearson type III (PT3):

$$\tau_4^{\mathrm{PT3}} = 0.12240 + 0.30115 \cdot \tau_3^2 + 0.95812 \cdot \tau_3^4 - 0.57488 \cdot \tau_3^6 + 0.19383 \cdot \tau_3^8 \quad (15)$$

and General of Extreme Values (GEV):

$$\tau_4^{\mathrm{GEV}} = 0.10701 + 0.11090 \cdot \tau_3 + 0.84838 \cdot \tau_3^2 - 0.06669 \cdot \tau_3^3 + SF \quad (16)$$

being: $$SF = 0.00567 \cdot \tau_3^4 - 0.04208 \cdot \tau_3^5 + 0.03763 \cdot \tau_3^6$$

Using the logarithms of the data in Equation (4), Equation (5), Equation (6), Equation (7), Equation (8), Equation (9), Equation (10) and Equation (11), we obtain the L logarithmic quotients and so we can use expression 15 to evaluate PDF Log–Pearson type III (LP3). Figure 1 shows the diagram of L moment quotients from Hosking and Wallis (1997).

Selection of the best PDF

One of the recent approaches for selecting the best PDF, to be used in FFA, consists of taking the sample values (τ_3 and τ_4) to the L quotient diagram and defining their nearness to one of the curves to obtain the best probabilistic model. To avoid subjectivity in selecting the PDF, evaluating the *minimum absolute distance* (*DA*) has been proposed, with the following expression (Yue & Hashino, 2007):

$$DA = \left|\tau_4[\tau_3^{obs}] - \tau_4^{obs}\right| \quad (17)$$

where τ_3^{obs} and τ_4^{obs} are the L quotients of asymmetry and kurtosis of the register or available series of annual maximum flows, and $\tau_4[\tau_3^{obs}]$ is the theoretical value of the L quotient of kurtosis calculated with each PDF (Equation (12), Equation (13), Equation (14), Equation (15) and Equation (16)) for the observed value of the asymmetry L quotient. A PDF with a value lower than the *DA* is the best for the data.

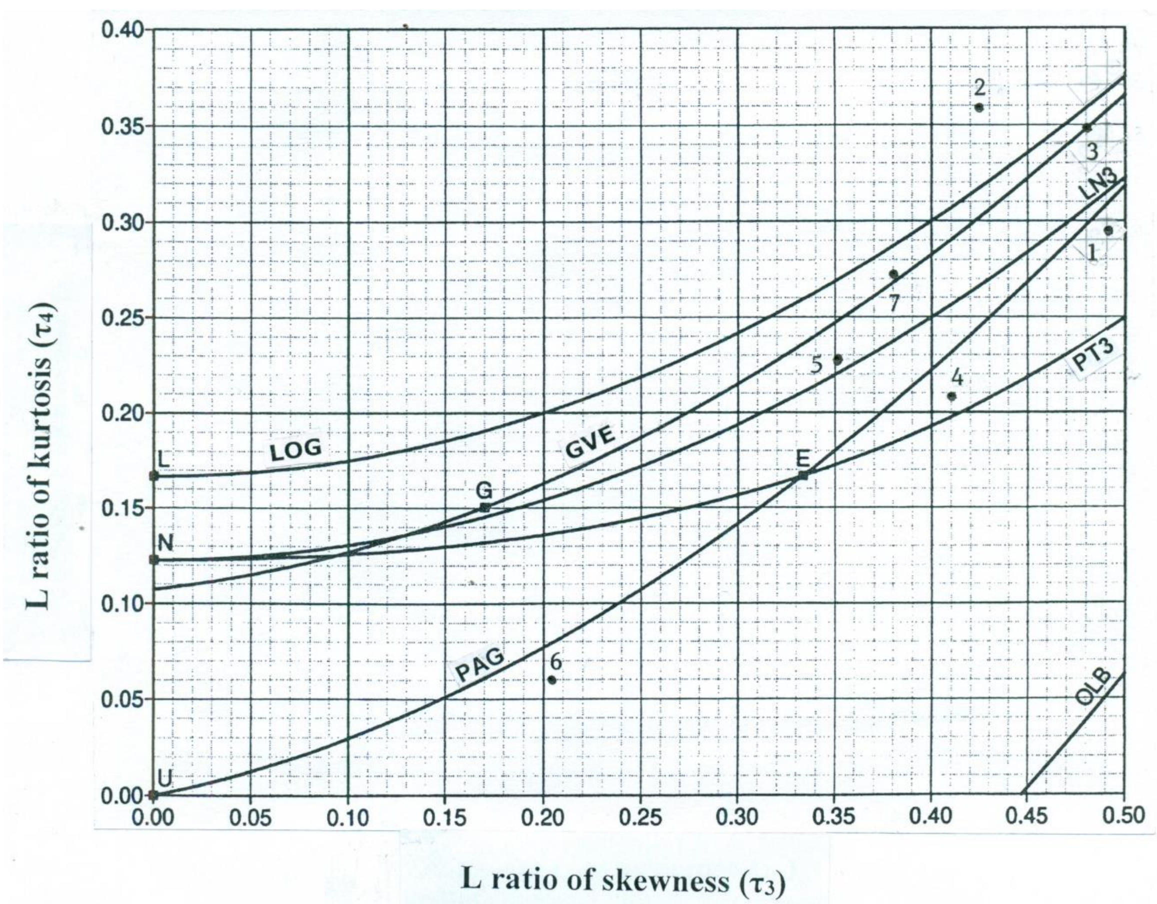

Figure 1. Diagram of L moment quotients, showing PDF of two and three fitting parameters (Hosking & Wallis, 1997).

Fitting source PDF

Exclusively, the LP3 distribution was applied with the moments method in the logarithmic dominion (WRC, 1977; Campos-Aranda, 2015); while Botto *et al*. (2017) fit it as a PT3 based on the logarithmic L quotients. The rest of the distributions from Table 1 were fit with the method of L moments, following the procedures described by Hosking and Wallis (1997). In all cases the standard error of fit (*SEF*) was evaluated.

Standard error of fit

The standard error of fit is the most common indicator for contrasting PDF against real data (Chai & Draxler, 2014). It was established in the mid–1970s (Kite, 1977), and has been applied in Mexico using the empirical formula of Weibull (Benson, 1962). Now, its application is recommended for use with the formula of Cunnane (Equation (18)), according to Stedinger (2017), leading to non−exceedance probabilities (*p*) approximately unbiased for many PDF; that is:

$$p = \frac{i-0.40}{n+0.20} \tag{18}$$

The expression for the standard error of fit is:

$$EEA = \left[\frac{\sum_{i=1}^{n}(Q_i-\hat{Q}_i)^2}{(n-np)}\right]^{1/2} \tag{19}$$

In which Q_i are annual maximum flows ordered from least to highest and whose number is n, and $\hat{Q}_i$ are the estimated maximum flows for the probability estimated with Equation (18) and the PDF that contrasts. *np* is the number of fitting parameters of the PDF, with three for the five to be applied.

Flood registers to be analyzed

To develop the numerical applications, we chose to process the seven largest registers (43 ≤ n ≤ 56) of hydrometric information processed by Campos-Aranda (2014) of Hydrological Region Num. 10 (Sinaloa), Mexico, corresponding to the Huites, Santa Cruz, Jaina, Naranjo, Acatitán, Zopilote and El Bledal stations. Table 2 cites two general characteristics and the L moment ratio values of the seven selected stations.

Table 2. Basin sizes, register amplitude and L moment ratios in the seven selected hydrometric stations of the Hydrological Region Num. 10 (Sinaloa), Mexico.

Station	***A* (km²)**	***n***	τ_3	τ_4	$\tau_3^{\ln}$	$\tau_4^{\ln}$
1. Huites	26 057	51	0.49086	0.29757	0.14918	0.14510
2. Santa Cruz	8 919	52	0.42451	0.35919	−0.08131	0.17382
3. Jaina	8 179	56	0.47970	0.34935	0.04559	0.18304
4. Naranjo	2 064	45	0.40967	0.20800	−0.03623	0.11706
5. Acatitán	1 884	43	0.35069	0.22836	−0.27398	0.20318
6. Zopilote	666	56	0.20325	0.06008	−0.20501	0.09740
7. El Bledal	371	56	0.38055	0.27257	−0.02134	0.15315

Figure 1 shows the graphic position of each register, according to its asymmetry and kurtosis L quotient. We deduced that the convenient PDF (because of their closeness) are the following: in Huites PAG, in Santa Cruz LOG, in Jaina GEV, in Naranjo PT3, in Acatitán LN3, in Zopilote PAG and in El Bledal GVE.

Results and analysis

Verification of register randomness

For the results of the FFA to be exact, the register of annual maximum flows to be processed must have been generated by a *stationary random process*, which implies that it has not changed over time. Therefore, the flood register should integrate independent data, exempt of deterministic components.

To test this, we applied the Wald–Wolfowitz Test, which is a non–parametric test used by Bobée and Ashkar (1991), Rao and Hamed (2000), and Meylan, Favre, and Musy (2012) to verify *independence* and *stationarity* in registers of annual maximum flows. Its application in the seven selected registers showed that they are integrated by independent random data.

The best PDF, according to minimum *DA*

Based on L moment ratios from Table 2, we used Equation (17) to find the two minimum *DA* and, in this way, define the two most suitable PDF in Table 3. The second option will be used in the Huites and Zopilote stations since Botto *et al*. (2017) did not establish the Generalized Pareto as the parent distribution in Table 1.

Table 3. Best two PDF, according to minimum Absolute Distance (*DA*) of the annual flood registers of the seven selected hydrometric stations of Hydrological Region Num. 10 (Sinaloa), Mexico.

Station	*DA*	PDF	*DA*	PDF
Huites	0.0113	PAG	0.0155	LP3
Santa Cruz	0.0423	LOG	0.0494	LP3
Jaina	0.0016	GVE	0.0091	LOG
Naranjo	0.0057	LP3	0.0106	PT3
Acatitán	0.0086	LN3	0.0189	GVE
Zopilote	0.0190	PAG	0.0393	LP3
El Bledal	0.0043	GVE	0.0148	LOG

Design predictions and their correction

Table 4 presents the estimated predictions with each of the most convenient PDF for the five return periods that Botto *et al.* (2017) analyzed. Moreover, their correction factors ($\hat{y}$) calculated with Equation (3) are shown, and finally, we have the corrected DF ($\hat{Q}^*_{Tr}$) due to hydrological uncertainty.

Table 4. Predictions (Q_{Tr}) in m^3/s obtained with the indicated PDF and corrected Design Floods ($\hat{Q}^*_{Tr}$) in the seven indicated hydrometric stations of Hydrological Region Num. 10 (Sinaloa), Mexico.

Station PDF *SEF* (m^3/s)	**Calculation (*n*)**	**Return period in years**				
		50	**100**	**200**	**500**	**1000**
Huites	Q_{Tr}	15 613	21 776	29 993	45 130	60 942
LP3	$\hat{y}$ (51)	0.050	0.081	0.130	0.244	0.392
957.1	$\hat{Q}^*_{Tr}$	16 394	23 540	33 892	56 142	84 831
Santa Cruz	Q_{Tr}	4 335	5 891	7 971	11 843	15 948
LOG	$\hat{y}$ (52)	0.008	0.015	0.030	0.075	0.149
277.5	$\hat{Q}^*_{Tr}$	4 370	5 979	8 210	12 731	18 324
Jaina	Q_{Tr}	4 419	6 101	8 367	12 614	17 148
GVE	$\hat{y}$ (56)	0.008	0.018	0.039	0.108	0.234

360.3	$\hat{Q}_{Tr}^{*}$	4 454	6 210	8 693	13 976	21 161
Naranjo	Q_{Tr}	3 090	3 973	4 976	6 499	7808
LP3	$\hat{y}$ (45)	0.056	0.090	0.145	0.273	0.439
136.2	$\hat{Q}_{Tr}^{*}$	3 263	4 331	5 698	8 273	11 236
Acatitán	Q_{Tr}	3 440	4 263	5 178	6 537	7 689
LN3	$\hat{y}$ (43)	0.020	0.035	0.062	0.130	0.229
150.5	$\hat{Q}_{Tr}^{*}$	3 509	4 412	5 499	7 387	9 450
Zopilote	Q_{Tr}	1 269	1 432	1 576	1 739	1 844
LP3	$\hat{y}$ (56)	0.046	0.074	0.119	0.223	0.359
67.1	$\hat{Q}_{Tr}^{*}$	1 327	1 538	1 764	2 127	2 506
El Bledal	Q_{Tr}	1 090	1 404	1 792	2 446	3 077
GVE	$\hat{y}$ (56)	0.008	0.018	0.039	0.108	0.234
64.0	$\hat{Q}_{Tr}^{*}$	1 099	1 429	1 862	2 710	3 797

We can observe in Table 4 that the smallest corrections ($\hat{y}$) in the *Tr* = 50 years were less than 1% in the Santa Cruz, Jaina and El Bledal stations, and the largest was 5.6% in Naranjo. In the *Tr* = 1000 years, the smallest correction is 14.9% in Santa Cruz and the largest rises to 43.9% in Naranjo. In general, the corrections of the LP3 distribution are the largest, while the smallest are those of the LOG model. This is most likely related to the density of the right tail (El Adlouni, Bobée, & Ouarda, 2008) of these two PDFs under precept application in the USA and England.

Conclusions

Because of its great simplicity, the correction method for Design Floods because of hydrological uncertainty, developed by Botto *et al*. (2017) and based on Equation (2) and Equation (3), the merit of this Note is exclusively that of dissemination.

In the seven numerical applications described, we use the largest registers of annual maximum flows of Hydrological Region Num. 10 (Sinaloa), Mexico, making use of the L quotient diagram and of minimum absolute distance to select objectively the best PDF for each available register.

The correction factors ($\hat{y}$) calculated with Equation (3) decrease as the size (n) of the register increases and grow with the value of the return period (Tr). In the Tr = 50 years, logically, we have the smallest corrections, which reach 5.6%. In contrast, in the Tr = 1 000 years, the corrections fluctuated from 14.9 to 43.9%.

When n and Tr values are equal, in Table 4 we observed that the $\hat{y}$ values are higher for the LP3 distribution and lower with LOG. For the other three PDF (GEV, LN3 and PT3), they are similar.

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